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D-branes at singularities: a bottom-up approach to the string embedding of the standard model

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Abstract: We propose a bottom-up approach to particle physics model building from string theory. We start this program by using configurations of type-IIB D3- and D7-branes located at singularities, combined appropriately to reproduce phenomenologically desirable features. We study in detail $\mathbb{Z}_N$ $\mathcal{N}=1,0$ orbifold singularities leading to SM or LR symmetric gauge group. The unique $\mathcal{N}=1$ supersymmetric twist leading to three generations is $\mathbb{Z}_3$, predicting $\sin^2\theta_W = 3/14 = 0.21$, and yielding the simplest semirealistic string models ever built. In the non-supersymmetric case, each $\mathbb{Z}_N$, $N>4$, leads to three-generation models, but the Weinberg angle is in general too small. One can obtain a large class of $D=4$ compact models by embedding the above structure into a Calabi Yau compactification. We construct explicit examples using $T^6\mathbb{Z}_3$ orbifolds and orientifolds, with additional antibrane hidden sectors, leading to gravity mediated supersymmetry breaking. We construct an explicit F-theory compactification with unbroken $\mathcal{N}=1$ supersymmetry,with a three-family LR sector.

Keywords: F-Theory, D-branes, Superstring Vacua, Brane Dynamics in Gauge Theories.

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1. Introduction

One of the important motivations in favour of string theory in the mid-eighties was the fact that it seemed to include in principle all the ingredients required to embed the observed standard model (SM) physics inside a fully unified theory with gravity. The standard approach when trying to embed the standard model into string theory has traditionally been an *top-down approach*. One starts from a string theory like e.g. the $E_8 \times E_8$ heterotic and reduces the number of dimensions, supersymmetries and the gauge group by an appropriate compactification leading to a massless spectrum as similar as possible to the SM. The paradigm of this approach [1] has been the compactification of the $E_8 \times E_8$ heterotic on a CY manifold with Euler characteristic $\chi = \pm 6$, leading to a three-generation E_6 model. Further gauge symmetry breaking may be achieved e.g. by the addition of Wilson lines [2] and a final breakdown of $D = 4$, $\mathcal{N} = 1$ supersymmetry is assumed to take place due to some field-theoretical non-perturbative effects [3]. Other constructions using compact orbifolds or fermionic string models follow essentially the same philosophy [4].

Although since 1995 our view of string theory has substantially changed, the concrete attempts to embed the SM into string theory have essentially followed the same traditional approach. This is the case for instance in the construction of a M-theory compactifications on CY$\times S^1/\mathbb{Z}_2$ [5, 6], or of F-theory compactifications on Calabi-Yau (CY) four-folds [7, 8], leading to new non-perturbative heterotic compactifications. This is still a top-down approach in which matching of the observed low-energy physics is expected to be achieved by searching among the myriads of CY three- or four-folds till we find the correct vacuum.[1]

The traditional top-down approach is in principle a reasonable possibility but it does not exploit fully some of the lessons we have learnt about string theory in recent years, most prominently the fundamental role played by different classes of p-branes (e.g. D-branes) in the structure of the full theory, and the important fact that they localize gauge interactions on their worldvolume without any need for compactification at this level. It also requires an exact knowledge of the complete geography of the compact extra dimensions (e.g. the internal CY space) in order to obtain the final effective action for massless modes.

[1] For recent attempts at semirealistic model building based on type-IIB orientifolds see refs. [9]–[16].

We know that, for example, type-IIB D3-branes have gauge theories with matter fields living in their worldvolume. These fields are localized in the four-dimensional world-volume, and their nature and behaviour depends only on the local structure of the string configuration in the vicinity of that four-dimensional subspace. Thus, as far as gauge interactions are concerned, it seems that the most sensible approach should be to look for D-brane configurations with world volume field theories resembling as much as possible the SM field theory, even before any compactification of the six transverse dimensions. Following this idea, in the present article we propose a *bottom-up approach* to the embedding of the SM physics into string theory. Instead of looking for particular CY compactifications of a $D = 10, 11, 12$ dimensional structure down to four-dimensions we propose to proceed in two steps:

(i) Look for *local* configurations of D-branes with worldvolume theories resembling the SM as much as possible. In particular we should search for a gauge group $SU(3) \times SU(2) \times U(1)$ but also for the presence of three chiral quark-lepton generations. Asking also for $D = 4$ $\mathcal{N} = 1$ unbroken supersymmetry may be optional, depending on what our assumptions about what solves the hierarchy problem are. At this level the theory needs no compactification and the D-branes may be embedded in the full ten-dimensional Minkowski space. On the other hand, gravity still remains ten-dimensional, and hence this cannot be the whole story.

(ii) The above *local* D-brane configuration may in general be part of a larger *global* model. In particular, if the six transverse dimensions are now compactified, one can in general obtain appropriate four-dimensional gravity with Planck mass proportional to the compactification radii.

This two-step process is illustrated in figure 1. An important point to realize is that, although taking the first step, i.e. finding a "SM brane configuration" may be relatively very restricted, the second step may be done possibly in myriads of manners. Some properties of the effective lagrangian (e.g. the gauge group, the number of generations, the normalization of coupling constants) will depend only on the *local* structure of the D-brane configuration. Hence many phenomenological questions can be addressed already at the level of step (i). On the other hand, other properties, like some Yukawa couplings and Kähler metrics, will be dependent on the structure of the full *global* model.

In this paper we present the first specific realizations of this *bottom-up* approach. We believe that the results are very promising and lead to new avenues for the understanding of particle theory applications of string theory. In particular, and concerning the two step bottom-up approach described above we find that:

(i) One can obtain simple configurations of type-IIB D3, D7 branes with world-volume theories remarkably close to the SM (or some Left-Right symmetric generalizations). They correspond to collections of D3/D7 branes located at orbifold singularities. The presence of additional branes beyond D3-branes (i.e. D7-branes) is dictated by tadpole cancellation conditions. Finding three quark-lepton generations

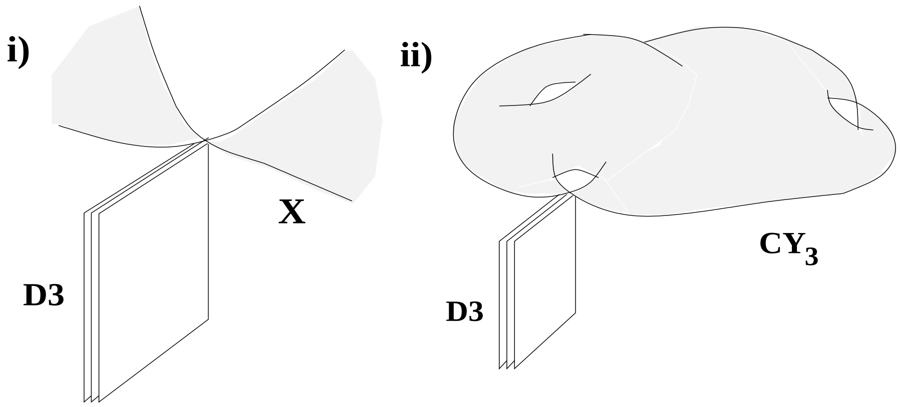

Figure 1: Pictorial representation of the bottom-up approach to the embedding of the standard model in string theory. In step (i) the standard model is realized in the world-volume of D3-branes sitting at a singular non-compact space X (in the presence of D7-branes, not depicted in the figure). In step (ii) this local configuration is embedded in a global context, like a compact Calabi-Yau threefold. Many interesting phenomenological issues depend essentially only on the local structure of X and are quite insensitive to the details of the compactification in step (ii). The global model may contain additional structures (like other branes or antibranes) not shown in the figure.

and $\mathcal{N}=1$ SUSY turns out to be quite restrictive leading essentially to $\mathbb{C}^3/\mathbb{Z}_3$ singularities or some variations (including related models with discrete torsion, $\mathbb{Z}_3$ orbifolds of the conifold singularity, or a non-abelian orbifold singularity based on the discrete group Δ_{27}). In these models extra gauged U(1)'s with triangle anomalies (cured by a generalized Green-Schwarz mechanism) are generically present but decouple at low energies. The appearance of the weak hypercharge $\mathrm{U}(1)_Y$ in these models is particularly elegant, corresponding to a unique universal linear combination which is always anomaly-free and yields the SM hypercharge assignments automatically. In the case of non-supersymmetric $\mathbb{Z}_N$ orbifold singularities, three quark-lepton generations may be obtained for any $N>4$, but the resulting weak angle tends to be too small.

(ii) These local "SM configurations" may be embedded into compact models yielding correct $D=4$ gravity. We construct specific compact type-IIB orbifold and orientifold models which contain subsectors given by the realistic D-brane configurations discussed above. In order to cancel global tadpoles (i.e, the total untwisted RR D-brane charges) one can add anti-D3 and/or anti-D7 branes. These antibranes are stuck at orbifold fixed points (in order to ensure stability against brane-antibrane annihilation) and lead to models with hidden sector gravity-induced supersymmetry breaking. Other compact models with unbroken $D=4$ $\mathcal{N}=1$ supersymmetry may be easily obtained in the more general framework of F-theory compactifications, and we construct an specific example of this type. In this approach the embedding of SM physics into F-theory is quite different from those followed previously: the interesting physics resides on D3-branes, rather than D7-branes.

As we comment above, some properties of the low-energy physics of the compact models will only depend on the local D3-brane configuration. That is for example the case of hypercharge normalization. The D-brane configurations leading to unbroken $\mathcal{N}=1$ SUSY predict a tree-level value for the weak angle $\sin^2\theta_W = 3/14 = 0.215$, different from the SU(5) standard result 3/8. This is compatible with standard logarithmic gauge coupling unification if the string scale is of order $M_s \propto 10^{11}$ GeV, as we discuss in the text. Other phenomenological aspects like Yukawa couplings depend not only on the singularity structure, but also on the particular form of the compactification. Notice in this respect that although the physics of the D3-branes will be dominated by the presence of the singularity, the D7-branes wrap subspaces in the compact space and are therefore more sensitive to its global structure.

The structure of this paper is as follows. In the following chapter we present some general results which will be needed in the remaining sections. We describe the general massless spectrum and couplings of type-IIB D3- and D7-branes on abelian orbifold singularities, both for the $\mathcal{N}=1$ and $\mathcal{N}=0$ cases. We also discuss the consistency conditions of these configurations (cancellation of RR twisted tadpoles), as well as the appearance of non-anomalous U(1) gauge symmetries in this class of theories. In chapter 3 we apply the formalism discussed in chapter 2 to the search of realistic three-generation D-brane configurations sitting on $\mathbb{R}^6/\mathbb{Z}_N$ singularities. We present specific simple $\mathcal{N}=1$ models leading to the SM gauge group with three quark-lepton generations. We also present an alternative $SU(3)\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ three generation model. We also discuss the case of non-SUSY $\mathbb{Z}_N$ singularities and present an specific three generation non-SUSY model based on a $\mathbb{Z}_5$ singularity.

Finally, we discuss different generalizations yielding also three quark-lepton generations. In particular we discuss orbifold singularities with discrete torsion, models based on non-abelian orbifolds, as well as some models based on certain non-orbifold singularities. Although the massless spectrum of these new possibilities is very similar to the models based on the $\mathbb{Z}_3$ singularity, some aspects like Yukawa couplings get modified, which may be interesting phenomenologically. We argue that locating the D3-branes on an orientifold (rather than orbifold) point does not lead to standard model configurations and hence is not very promising. Finally we discuss non-supersymmetric models constructed using branes and antibranes.

In chapter 4 we proceed to the second step in our approach and embed the realistic D3/D7 configurations found in the previous chapters into a compact space. We present examples based on type-IIB orbifolds and orientifolds. As we mentioned above, in this models the global RR charges may be canceled by the addition of anti-D-branes which are trapped at the fixed points. Some of these models are T-dual to the models recently studied in [14, 16]. We also discuss the construction of models with unbroken $D=4$, $\mathcal{N}=1$ SUSY by considering F-theory compactifications with the SM embedded on D3-branes, and construct an specific example. In chapter 5 we briefly discuss some general phenomenological questions, like gauge coupling unification and Yukawa couplings. We leave our final comments and outlook for chapter 6. In order not to obstruct continuity in the reading with many details, we have five appendices. The first four give the details of each of the generalizations mentioned in section 3.6. The last appendix deals with the issue of T-duality on some of the compact models in section 4.

2. Three-branes and seven-branes at abelian orbifold singularities

In this section we introduce the basic formalism to compute the spectrum and interactions on the world-volume of D3-branes at $\mathbb{R}^6/\mathbb{Z}_N$ singularities.[2] These have been discussed in [17]–[21], but our treatment is more general in that we allow the presence of D7-branes. We also discuss several aspects of these field theories, to be used in the remaining sections.

Before entering the construction, we would like to explain our interest in placing the D3-branes on top of singularities. The reason is that D3-branes sitting at smooth points in the transverse dimensions lead to $\mathcal{N}=4$ supersymmetric field theories. The only known way to achieve chirality in this framework is to locate the D3-branes at singularities, the simplest examples being $\mathbb{R}^6/\mathbb{Z}_N$ orbifold singularities. Another important point in our approach is that we embed all gauge interactions in D3-branes. Our motivation for this is the fact that the appearance of SM fermions in three copies is difficult to achieve if color and weak interactions live in e.g. D3- and D7-branes, respectively (as will be manifest from the general spectra below).

2.1 Brane spectrum

We start by considering the case of a generic, not necessarily supersymmetric, singularity. Later on we discuss the specific $\mathcal{N}=0$ non supersymmetric and $\mathcal{N}=1$ supersymmetric realizations. Consider a set of n D3-branes at a $\mathbb{R}^6/\Gamma$ singularity with $\Gamma \subset \mathrm{SU}(4)$ where, for simplicity, we take $\Gamma = \mathbb{Z}_N$. Before the projection, the world-volume field theory on the D3-branes is a $\mathcal{N}=4$ supersymmetric $\mathrm{U}(n)$ gauge theory. In $\mathcal{N}=1$ language, it contains $\mathrm{U}(n)$ vector multiplets, and three adjoint chiral multiplets Φ^r, $r=1,2,3$, with interactions determined by the superpotential

$$W = \sum_{r,s,t} \epsilon_{rst}\,\mathrm{Tr}(\Phi^r\Phi^s\Phi^t)\,. \tag{2.1}$$

In terms of component fields, the theory contains $\mathrm{U}(n)$ gauge bosons, four adjoint fermions transforming in the **4** of the $\mathrm{SU}(4)_R$ $\mathcal{N}=4$ R-symmetry group, and six adjoint real scalar fields transforming in the **6**.

[2] Other cases, like $\mathbb{Z}_N\times\mathbb{Z}_M$ orbifolds can be studied analogously, and we will skip their discussion.

The $\mathbb{Z}_N$ action on fermions is given by a matrix

$$\mathbf{R_4} = \text{diag}\left(e^{2\pi i a_1/N}, e^{2\pi i a_2/N}, e^{2\pi i a_3/N}, e^{2\pi i a_4/N}\right) \tag{2.2}$$

with $a_1+a_2+a_3+a_4 = 0 \bmod N$. The action of $\mathbb{Z}_N$ on scalars can be obtained from the definition of the action on the $\mathbf{4}$, and it is given by the matrix

$$\mathbf{R_6} = \text{diag}\left(e^{2\pi i b_1/N}, e^{-2\pi i b_1/N}, e^{2\pi i b_2/N}, e^{-2\pi i b_2/N}, e^{2\pi i b_3/N}, e^{-2\pi i b_3/N}\right) \tag{2.3}$$

with $b_1 = a_2 + a_3$, $b_2 = a_1 + a_3$, $b_3 = a_1 + a_2$. Scalars can be complexified, the action on them being then given by $\mathbf{R}_{esc} = \text{diag}(e^{2\pi i b_1/N}, e^{2\pi i b_2/N}, e^{2\pi i b_3/N})$. Notice that, since scalars have the interpretation of brane coordinates in the transverse space, eq. (2.3) defines the action of $\mathbb{Z}_N$ on $\mathbb{R}^6$ required to form the quotient $\mathbb{R}^6/\mathbb{Z}_N$.

The action of the $\mathbb{Z}_N$ generator θ must be embedded on the Chan-Paton indices. In order to be more specific we consider the general embedding given by the matrix

$$\gamma_{\theta,3} = \text{diag}\left(\text{I}_{n_0}, e^{2\pi i/N}\text{I}_{n_1}, \ldots, e^{2\pi i(N-1)/N}\text{I}_{n_{N-1}}\right), \tag{2.4}$$

where I_{n_i} is the $n_i \times n_i$ unit matrix, and $\sum_i n_i = n$. The theory on D3-branes at the $\mathbb{R}^6/\mathbb{Z}_N$ singularity is obtained by keeping the states invariant under the combined (geometrical plus Chan-Paton) $\mathbb{Z}_N$ action [17, 18]. World-volume gauge bosons correspond to open string states in the NS sector, of the form $\lambda\psi^{\mu}_{-1/2}|0\rangle$, with μ along the D3-brane world-volume, and λ the Chan-Paton wavefunction. The projection for gauge bosons is then given by

$$\lambda = \gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1}. \tag{2.5}$$

The projection for each of the three complex scalars, $\lambda\Psi^{r}_{-1/2}|0\rangle$ (with $r = 1,2,3$ labeling a complex plane transverse to the D3-brane) is

$$\lambda = e^{-2\pi i b_r/N}\gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1}. \tag{2.6}$$

The four fermions in the D3-brane world-volume, labeled by $\alpha = 1,\ldots,4$ are described by string states in the R sector, of the form $\lambda|s_1,s_2,s_3,s_4\rangle$, with $s_i = \pm 1/2$ and $\sum_i s_i =$ odd. The projection for left-handed fermions, $s_4 = -1/2$, leads to

$$\lambda = e^{2\pi i a_\alpha/N}\gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1}. \tag{2.7}$$

The final spectrum in the 33 sector is

$$\begin{aligned}
\text{Vectors} \quad & \prod_{i=0}^{N-1} \text{U}(n_i), \\
\text{Complex Scalars} \quad & \sum_{r=1}^{3}\sum_{i=0}^{N-1} (n_i, \overline{n}_{i-b_r}), \\
\text{Fermions} \quad & \sum_{\alpha=1}^{4}\sum_{i=0}^{N-1} (n_i, \overline{n}_{i+a_\alpha}),
\end{aligned} \tag{2.8}$$

where subindices will be understood modulo N throughout the paper. The interactions are obtained by keeping the surviving fields in the interactions of the original $\mathcal{N}=4$ theory. Notice that the spectrum is, generically, non supersymmetric. Instead, when $b_1+b_2+b_3=0$, we have $a_4=0$ and the $\mathbb{Z}_N$ action is in SU(3). This case corresponds to a supersymmetric singularity. The fermions with $\alpha=4$ transforming in the adjoint representation of $\mathrm{U}(n_i)$ become gauginos, while the other fermions transform in the same bifundamental representations as the complex scalars. The different fields fill out complete vector and chiral multiplets of $\mathcal{N}=1$ supersymmetry.

In general, we would also like to include D7-branes in the configuration. Let us center on D7-branes transverse to the third complex plane Y_3, denoted $\mathrm{D7}_3$-branes in what follows. Open strings in the 37_3 and 7_33 sectors contribute new fields in the D3-brane world-volume. In the R sector, there are fermion zero modes in the NN and DD directions Y_4, Y_3. Such states are labeled by $\lambda|s_3;s_4\rangle$, with $s_3=s_4=\pm 1/2$, where s_4 defines the spacetime chirality. The projection for left-handed fermions $s_4=-1/2$ are

$$\lambda_{37_3}=e^{i\pi b_3/N}\gamma_{\theta,3}\lambda\gamma_{\theta,7_3}^{-1}\,,\qquad \lambda_{7_33}=e^{i\pi b_3/N}\gamma_{\theta,7_3}\lambda\gamma_{\theta,3}^{-1}\,. \tag{2.9}$$

Scalars arise from the NS sector, which contains fermion zero modes in the DN directions (Y_1, Y_2). States are labeled as $\lambda|s_1,s_2\rangle$, with $s_1=s_2=\pm 1/2$. The projections for $\lambda|1/2,1/2\rangle$ are

$$\lambda_{37_3}=e^{-i\pi(b_1+b_2)/N}\gamma_{\theta,3}\lambda\gamma_{\theta,7_3}^{-1}\,,\qquad \lambda_{7_33}=e^{-i\pi(b_1+b_2)/N}\gamma_{\theta,7_3}\lambda\gamma_{\theta,3}^{-1}\,. \tag{2.10}$$

We can give the resulting spectrum quite explicitly. Let us consider the Chan-Paton embedding

$$\begin{aligned}
\gamma_{\theta,7_3}&=\mathrm{diag}\left(\mathrm{I}_{u_0},e^{2\pi i/N}\mathrm{I}_{u_1},\ldots,e^{2\pi i(N-1)/N}\mathrm{I}_{u_{N-1}}\right) && \text{for } b_3=\text{even}\,,\\
\gamma_{\theta,7_3}&=\mathrm{diag}\left(e^{\pi i\frac{1}{N}}\mathrm{I}_{u_0},e^{2\pi i\frac{3}{N}}\mathrm{I}_{u_1},\ldots,e^{2\pi i\frac{2N-1}{N}}\mathrm{I}_{u_{N-1}}\right) && \text{for } b_3=\text{odd}\,.
\end{aligned} \tag{2.11}$$

The resulting spectrum is

$$\begin{aligned}
b_3=\text{even}\longrightarrow\ &\text{Fermions} && \sum_{i=0}^{N-1}\left[\left(n_i,\overline{u}_{i+\frac{1}{2}b_3}\right)+\left(u_i,\overline{n}_{i+\frac{1}{2}b_3}\right)\right],\\
&\text{Complex Scalars} && \sum_{i=0}^{N-1}\left[\left(n_i,\overline{u}_{i-\frac{1}{2}(b_1+b_2)}\right)+\left(u_i,\overline{n}_{i-\frac{1}{2}(b_1+b_2)}\right)\right],\\
b_3=\text{odd}\longrightarrow\ &\text{Fermions} && \sum_{i=0}^{N-1}\left[\left(n_i,\overline{u}_{i+\frac{1}{2}(b_3-1)}\right)+\left(u_i,\overline{n}_{i+\frac{1}{2}(b_3+1)}\right)\right],\\
&\text{Complex Scalars} && \sum_{i=0}^{N-1}\left[\left(n_i,\overline{u}_{i-\frac{1}{2}(b_1+b_2+1)}\right)+\left(u_i,\overline{n}_{i-\frac{1}{2}(b_1+b_2-1)}\right)\right].
\end{aligned} \tag{2.12}$$

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The computation is identical for other D7_r-branes, transverse to the r-th complex plane, i.e. with world-volume defined by the equation $Y_r = 0$. Notice that for a general twist, D7-branes with world-volume $\sum_{r=1}^{3} \beta_r Y_r = 0$, with arbitrary complex coefficients β_r, are not consistent with the orbifold action (more precisely, they are not invariant under the orbifold action, and suitable $\mathbb{Z}_N$ images should be included). For twists with several equal eigenvalues, such D7-branes are possible (see footnote 5).

Notice that in the non-compact setting, fields in the 77 sector are non-dynamical from the viewpoint of the D3-brane world-volume field theory. For instance, 77 gauge groups correspond to global symmetries, and 77 scalars act as parameters of the D3-brane field theory. Only after compactification of the transverse space, as in the models discussed in section 4, 77 fields become four-dimensional, and should be treated on an equal footing with 33 and 37, 73 fields.

We conclude this section by restricting the above results to the case of singularities $\mathbb{C}^3/\mathbb{Z}_N$ preserving $\mathcal{N}=1$ supersymmetry on the D3-brane world-volume. That is, for $a_4 = 0$, and hence $a_1 + a_2 + a_3 = 0 \bmod N$. The spectrum is given by

$$\begin{array}{lll}
\mathbf{33} & \text{Vector mult.} & \prod_{i=0}^{N-1} \mathrm{U}(n_i)\,, \\
 & \text{Chiral mult.} & \sum_{i=0}^{N-1} \sum_{r=1}^{3} (n_i, \overline{n}_{i+a_r})\,, \\
\mathbf{37_3}, \mathbf{7_33} & \text{Chiral mult.} & \sum_{i=0}^{N-1} \left[\left(n_i, \overline{u}_{i-\frac{1}{2}a_3}\right) + \left(u_i, \overline{n}_{i-\frac{1}{2}a_3}\right) \right] \quad a_3 \text{ even}\,, \\
 & & \sum_{i=0}^{N-1} \left[\left(n_i, \overline{u}_{i-\frac{1}{2}(a_3+1)}\right) + \left(u_i, \overline{n}_{i-\frac{1}{2}(a_3-1)}\right) \right] \quad a_3 \text{ odd}\,.
\end{array} \tag{2.13}$$

We will denote $\Phi^r_{i,i+a_r}$ the 33 chiral multiplet in the representation $(n_i, \overline{n}_{i+a_r})$. We also denote (assuming $a_3 =$ even for concreteness) $\Phi^{(37_3)}_{i,i-\frac{1}{2}a_3}$, $\Phi^{(7_33)}_{i,i-\frac{1}{2}a_3}$ the 37_3 and 7_33 chiral multiplets in the $(n_i, \overline{u}_{i-\frac{1}{2}a_3})$, $(u_i, \overline{n}_{i-\frac{1}{2}a_3})$. With this notation, the interactions are encoded in the superpotential

$$\begin{aligned}
W = & \sum_{r,s,t=1}^{3} \epsilon_{rst} \operatorname{Tr}\left(\Phi^r_{i,i+a_r} \Phi^s_{i+a_r,i+a_r+a_s} \Phi^t_{i+a_r+a_s,i}\right) + \\
& + \sum_{i=0}^{N-1} \operatorname{Tr}\left(\Phi^3_{i,i+a_3} \Phi^{(37_3)}_{i+a_3,i+\frac{1}{2}a_3} \Phi^{(7_33)}_{i+\frac{1}{2}a_3,i}\right).
\end{aligned}$$

2.2 Anomaly and tadpole cancellation

With the fermionic spectrum at hand we can proceed to compute non-abelian anomalies and establish the constraints for consistent anomaly-free theories. Moreover, such constraints can be rephrased in terms of twisted tadpole cancellation conditions [22, 23]. Let us address the computation of the non-abelian anomaly for $\mathrm{SU}(n_i)$, in a case with D7$_r$-branes, $r = 1, 2, 3$. Let us assume b_r = even for concreteness, and denote by u_j^r the number of entries with phase $e^{2\pi i j/N}$ in $\gamma_{\theta,7_r}$. The $\mathrm{SU}(n_j)$ non-abelian anomaly cancellation conditions are

$$\sum_{\alpha=1}^{4} (n_{i+a_\alpha} - n_{i-a_\alpha}) + \sum_{r=1}^{3} \left(u^r_{i+\frac{1}{2}b_r} - u^r_{i-\frac{1}{2}b_r} \right) = 0 \,. \tag{2.14}$$

These conditions are equivalent to the consistency conditions of the string theory configuration, namely cancellation of RR twisted tadpoles. To make this explicit, we use

$$n_j = \frac{1}{N} \sum_{k=1}^{N} e^{-2\pi i k j/N} \,\mathrm{Tr}\, \gamma_{\theta^k,3} \,; \qquad u_j^r = \frac{1}{N} \sum_{k=1}^{N} e^{-2\pi i k j/N} \,\mathrm{Tr}\, \gamma_{\theta^k,7_r} \tag{2.15}$$

and substitute in (2.14). We obtain

$$\frac{2i}{N} \sum_{k=1}^{N} e^{-2\pi i j k/N} \left[\sum_{\alpha=1}^{4} \sin\left(\frac{2\pi k a_\alpha}{N} \right) \mathrm{Tr}\, \gamma_{\theta^k,3} + \sum_{r=1}^{3} \sin\left(\frac{\pi k b_r}{2} \right) \mathrm{Tr}\, \gamma_{\theta^k,7_r} \right] = 0 \,. \tag{2.16}$$

Using the identity

$$\sum_{\alpha=1}^{4} \sin\left(\frac{2\pi k a_\alpha}{N} \right) = 4 \prod_{r=1}^{3} \sin\left(\frac{\pi k b_r}{N} \right) \tag{2.17}$$

the Fourier-transformed anomaly cancellation condition is recast as

$$\left[\prod_{r=1}^{3} 2 \sin\left(\frac{\pi k b_r}{N} \right) \right] \mathrm{Tr}\, \gamma_{\theta^k,3} + \sum_{r=1}^{3} 2 \sin\left(\frac{\pi k b_r}{N} \right) \mathrm{Tr}\, \gamma_{\theta^k,7_r} = 0 \,. \tag{2.18}$$

These are in fact the twisted tadpole cancellation conditions. Notice that the contributions from the different disk diagrams are weighted by suitable sine factors, which arise from integration over the string center of mass in NN directions.

2.3 Structure of $\mathrm{U}(1)$ anomalies and non-anomalous $\mathrm{U}(1)$'s

An important property of systems of D3-branes at singularities leading to chiral world-volume theories is the existence of mixed $\mathrm{U}(1)$-nonabelian gauge anomalies. These field theory anomalies are canceled by a generalized Green-Schwarz mechanism mediated by closed string twisted modes [24] (see [25, 17, 26] for a similar mechanism in six dimensions).

Consider the generically non-supersymmetric field theory constructed from D3- and D7$_r$-branes at a $\mathbb{Z}_N$ singularity, with spectrum given in (2.8) and (2.12). Assuming b_r = even for concreteness, the mixed anomaly between the j-th $\mathrm{U}(1)$ (that within $\mathrm{U}(n_j)$) and $\mathrm{SU}(n_l)$ is

$$A_{jl} = \frac{1}{2} n_j \sum_{\alpha=1}^{4} (\delta_{l,j+a_\alpha} - \delta_{l,j-a_\alpha}) + \frac{1}{2}\delta_{j,l}\left[\sum_{\alpha=1}^{4}(n_{j+a_\alpha} - n_{j-a_\alpha}) + \sum_{r=1}^{3}\left(u^r_{j-\frac{1}{2}b_r} - u^r_{j+\frac{1}{2}b_r}\right)\right].$$

The anomaly is not present if n_j or n_l vanish. After using the cancellation of cubic non-abelian anomalies (2.14) (i.e. the tadpole cancellation conditions), the remaining piece is given by

$$A_{jl} = \frac{1}{2} n_j \sum_{\alpha=1}^{4} (\delta_{l,j+a_\alpha} - \delta_{l,j-a_\alpha})\,. \tag{2.19}$$

This anomaly can be rewritten as

$$A_{jl} = \frac{-i}{2N}\sum_{k=1}^{N-1}\left[n_j \exp\left(\frac{2i\pi kj}{N}\right)\exp\left(\frac{-2i\pi kl}{N}\right)\prod_{r=1}^{3} 2\sin\left(\frac{\pi k b_r}{N}\right)\right] \tag{2.20}$$

which makes the factorized structure of the anomaly explicit. The anomaly is canceled by exchange of closed string twisted modes [24], which have suitable couplings to the gauge fields on the D-brane world-volume [17, 18, 27, 28].

An important fact is that anomalous U(1)'s get a tree-level mass of the order of the string scale [29], and therefore do not appear in the low-energy field theory dynamics on the D3-brane world-volume. It is therefore interesting to discuss the existence of non-anomalous U(1)'s and their structure. Concretely, we consider linear combinations of the U(1) generators

$$Q_c = \sum_{j=0}^{N-1} c_j \frac{Q_{n_j}}{n_j} \tag{2.21}$$

(we take $c_j = 0$ if $n_j = 0$). The condition for U(1)'s free of $Q_c - \mathrm{SU}(n_l)^2$ anomalies reads

$$\frac{1}{2}\sum_{\alpha=1}^{4}\sum_{j=0}^{N-1} c_j(\delta_{l,j+a_\alpha} - \delta_{l,j-a_\alpha}) = \sum_{\alpha=1}^{4}(c_{l-a_\alpha} - c_{l+a_\alpha}) = 0 \tag{2.22}$$

for all $l = 0, \ldots, N-1$. It is clear that $c_j =$ const. lead to an anomaly free combination

$$Q_{\mathrm{diag.}} = \sum_{i=0}^{N-1} \frac{Q_{n_i}}{n_i}\,. \tag{2.23}$$

This generically non-anomalous U(1) plays a prominent role in the realistic models of section 3. In general (2.22) gives $N-1$ independent conditions for the N unknowns c_j, and (2.23) is the only non-trivial solution (as long as no n_j vanishes). In certain cases, however, the number of independent equations may be smaller, and additional non-anomalous U(1)'s appear. In order to compute the number of independent equations we rewrite the unknowns c_j in terms of the new variables $r_k = \sum_{j=0}^{N-1} e^{2i\pi kj/N} c_j$. The original variables are given by

$$c_j = \frac{1}{N}\sum_{k=0}^{N-1} e^{-2\pi ikj/N} r_k \,. \tag{2.24}$$

By replacing in (2.22) above we find $\sum_{\alpha=1}^{4} \sin(2\pi k a_\alpha/N) r_k = 0$ and, by using the identity (2.17) we obtain

$$\left[\prod_{r=1}^{3} \sin\left(\frac{\pi k b_r}{N}\right)\right] r_k = 0\,. \tag{2.25}$$

Thus, we have managed to diagonalize the matrix corresponding to eq. (2.22). We see that, besides the diagonal solution (corresponding to $r_i = 0$ for $i = 0, \ldots, N-1$) other non-trivial solutions are possible whenever there exist twists θ^k that leave, at least, one unrotated direction. In other words, the rank of the matrix is given by the number of twists rotating all complex planes. It is possible to describe explicitly the non-anomalous U(1)'s, as follows. Let us consider a $\mathbb{Z}_M$ subgroup, generated by a certain twist θ^p, leaving e.g. the third complex plane fixed, hence $pa_1 = -pa_2$ and $pa_3 = -pa_4$. For each value of $I = 1, \ldots, M-1$, there is one non-anomalous U(1), defined by $c_j = \delta_{pj,I}$. This satisfies the condition (2.22) by a cancellation of contributions from different values of α. Hence we obtain one additional non-anomalous U(1) per twist leaving some complex plane fixed.[3]

Let us consider some explicit examples. For instance, for $\mathbb{Z}_3$ with twist $v = \frac{1}{3}(1, 1, -2)$, equation (2.22) leads to $b_0 = b_1 = b_2$ and the only anomaly free combination is (2.23). Consequently, it is not possible to have an anomaly free U(1), unless all $n_i \neq 0$. This will always be the case for $\mathbb{Z}_N$ orbifold actions (supersymmetric or not) without twists with fixed planes. Consequently, and as we discuss further in section 3, if we are interested in gauge groups similar to the Standard Model one, with an anomaly free (hypercharge) U(1), then all n_i's in (2.4) should be non vanishing.

The situation is different when subgroups with fixed planes exist. For instance, consider the $\mathbb{Z}_6 \equiv \frac{1}{6}(1, 1, -2)$ example. The **33** spectrum reads

$$\mathrm{U}(n_0) \times \mathrm{U}(n_1) \times \mathrm{U}(n_2) \times \mathrm{U}(n_3) \times \mathrm{U}(n_4) \times \mathrm{U}(n_5) \times \mathrm{U}(n_6)$$

$$2[(n_0, \overline{n}_1) + (n_1, \overline{n}_2) + (n_2, \overline{n}_3) + (n_3, \overline{n}_4) + (n_4, \overline{n}_5) + (n_5, \overline{n}_0) + (n_3, \overline{n}_0)] \tag{2.26}$$

$$+(n_0, \overline{n}_4) + (n_2, \overline{n}_0) + (n_4, \overline{n}_2) + (n_1, \overline{n}_5) + (n_3, \overline{n}_1) + (n_5, \overline{n}_3) \tag{2.27}$$

[3]There are arguments suggesting these additional non-anomalous U(1)'s are nevertheless massive due to their mixing with closed string twisted modes [30]. This observation however, would not change our analysis in the following sections, since our model building involves only the diagonal combination (2.23), for which such mixing vanishes.

and the anomaly matrix (2.19) is given by

$$2T_{ij}^{\alpha\beta} = \begin{pmatrix} 0 & 2n_0 & -n_0 & 0 & n_0 & -2n_0 \\ -2n_1 & 0 & 2n_1 & -n_1 & 0 & n_1 \\ n_2 & -2n_2 & 0 & 2n_2 & -n_2 & 0 \\ 0 & n_3 & -2n_3 & 0 & 2n_3 & -n_3 \\ -n_4 & 0 & n_4 & -2n_4 & 0 & 2n_4 \\ 2n_5 & -n_5 & 0 & n_5 & -2n_5 & 0 \end{pmatrix} . \tag{2.28}$$

The search for anomaly-free combinations leads to eq. (2.22) above, which has two non-trivial independent solutions. In fact, (2.24) becomes $c_i = r_0 + (-1)^i r_3$, namely $c_0 = c_2 = c_4$ and $c_1 = c_3 = c_5$ indicating that two anomaly-free abelian factors can be present. In particular, by choosing $n_1 = n_3 = n_5 = 0$ and $n_4 = 3, n_2 = 2, n_0 = 1$ we obtain a field theory with Standard Model group $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ and one generation $(\mathbf{3}, \overline{\mathbf{2}})_{1/6} + (\mathbf{1}, \mathbf{2})_{1/2} + (\overline{\mathbf{3}}, \mathbf{1})_{-2/3}$, with subscripts giving U(1) charges (fields in a suitable 37 sector should complete this to a full SM generation).

3. Particle models from branes at singularities

In this section we discuss the embedding of the Standard Model (and related Left-Right symmetric extensions) in systems of D3-branes at $\mathbb{C}^3/\mathbb{Z}_N$ singularities, with $\mathbb{Z}_N \subset \mathrm{SU}(3)$. We also discuss possible extensions to more general cases.

3.1 Number of generations and hypercharge

Let us start by recalling the structure of the field theory on D3-branes at a $\mathbb{C}^3/\mathbb{Z}_N$ singularity, defined by the twist $v = (a_1, a_2, a_3)/N$. It has $\mathcal{N} = 1$ supersymmetry, and the following field content: there are vector multiplets with gauge group $\prod_{i=0}^{N-1} \mathrm{U}(n_i)$, and $3N$ $\mathcal{N} = 1$ chiral multiplets $\Phi^r_{i,i+a_r}$, $i = 0, \ldots, N-1$, $r = 1, 2, 3$, transforming in the representation $(n_i, \overline{n}_{i+a_r})$. The interactions are encoded in the superpotential

$$W = \epsilon_{rst} \operatorname{Tr} \left(\Phi^r_{i,i+a_r} \Phi^s_{i+a_r,i+a_r+a_s} \Phi^t_{i+a_r+a_s,i} \right) . \tag{3.1}$$

In general, the configurations will also include D7-branes, but for the moment we center of general features in the **33** sector.

We are interested in constructing theories similar to the standard model or some simple extension thereof. In particular, we will be interested in constructing models which explicitly contain an $\mathrm{SU}(3) \times \mathrm{SU}(2)$ factor, to account for color and weak interactions. Besides simplicity, this choice has the additional advantage (discussed in detail below) that a non-anomalous U(1) leading to correct hypercharge assignments arises naturally.

Hence we consider models in which two factors, which without loss of generality we take $\mathrm{U}(n_0)$ and $\mathrm{U}(n_j)$, are actually U(3), U(2). Since the matter content contains only bi-fundamental representations, the number of generations is given by the number of left-handed quarks, i.e. fields in the representation $(\mathbf{3},\mathbf{2})$. This is given by the number of twist eigenvalues a_r equal to j (minus the number of a_r equal to $-j$), which is obviously at most three, this maximum value occurring only for the $\mathbb{C}^3/\mathbb{Z}_3$ singularity, with twist $v=(1,1,-2)/3$. For this reason, this singularity will play a prominent role in our forthcoming models.

Before centering on that concrete case, it will be useful to analyze the issue of hypercharge. In order to obtain a theory with standard model gauge group, one needs the presence of at least one non-anomalous U(1) to play the role of hypercharge. Happily, our analysis in section 2.3 has shown that, as long as no n_i vanishes, D3-branes at singularities always have at least one non-anomalous U(1) generated by $Q_{\rm diag}$ in (2.23). Therefore, a possibility to obtain the standard model gauge group, with no additional non-abelian factors would be to consider models with group[4] $U(3)\times U(2)\times U(1)^{N-2}$. In the generic case, only the diagonal combination

$$Q_{\rm diag}=-\left(\frac{1}{3}Q_3+\frac{1}{2}Q_2+\sum_{s=1}^{N-2}Q_1^{(s)}\right) \tag{3.2}$$

will be non anomalous (the overall minus sign is included for later convenience). In a generic orbifold all other $N-1$ additional U(1) factors will be anomalous and therefore massive, with mass of the order of the string scale. Of course, the fact that we have a non-anomalous U(1) does not guarantee it has the right properties of hypercharge. Quite surprisingly this is precisely the case for (3.2). For instance, fields transforming in the $(\mathbf{3},\mathbf{2})$ representation have $Q_{\rm diag}$ charge $-1/3+1/2=1/6$, as corresponds to left-handed quarks. Fields transforming in the $(\mathbf{3},\mathbf{1})$ (necessarily with charge -1 under one of the $Q_1^{(s)}$ generators) have a $Q_{\rm diag.}$ charge $-1/3+1=-2/3$, as corresponds to right-handed U quarks, etc. Analysis of the complete spectrum requires information about the D7-brane sector, and is postponed until the construction of explicit examples in section 3.3. Notice that correct hypercharge assignments would not be obtained had our starting point been e.g. $SU(4)\times SU(2)$, hence our interest in the $SU(3)\times SU(2)$ structure.

It is worth noticing that normalization of this hypercharge U(1) depends on N. In fact, by normalizing U(n) generators such that $\mathrm{Tr}\,T_a^2=1/2$ the normalization of Y generator is fixed to be

$$k_1=5/3+2(N-2)\,. \tag{3.3}$$

This amounts to a dependence on N in the Weinberg angle, namely

$$\sin^2\theta_W=\frac{1}{k_1+1}=\frac{3}{6N-4}\,. \tag{3.4}$$

[4] Such choice for the Chan-Paton embedding for D3-branes leads in general to non-vanishing tadpoles, which can be canceled by the introduction of D7-branes. We leave their discussion for the explicit examples below.

Thus the weak angle decreases as N increases. Notice that the SU(5) result 3/8 is only obtained for a $\mathbb{Z}_2$ singularity. However in that case the (33) spectrum is necessarily vector-like and hence one cannot reproduce the SM spectrum.

We will also be interested in constructing Left-Right symmetric extensions of the standard model. In particular, we consider gauge groups with a factor $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$, which are obtained by choosing suitable values for three of the entries n_i in the Chan-Paton embedding. As above, the corresponding tadpole must be canceled by additional D7-branes, whose details we postpone for the moment. The number of generations is again given by the number of representations $(\mathbf{3},\mathbf{2},\mathbf{1})$, and is equal to three only for the $\mathbb{Z}_3$ orbifold.

To reproduce hypercharge after the breaking of the right-handed SU(2) factor, an essential ingredient is the existence of a non-anomalous $(B{-}L)$ U(1) in the theory. In order to obtain at least one non-anomalous U(1) in the D3-branes, we are led to consider models with group $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(2) \times \mathrm{U}(1)^{N-3}$. Generically, only the diagonal combination

$$Q_{\rm diag} = -2\left(\frac{1}{3}Q^{(3)} + \frac{1}{2}Q^{(2_L)} + \frac{1}{2}Q^{(2_R)} + \sum_{s=1}^{N-3} Q_s^{(1)}\right) \qquad (3.5)$$

(the overall factor is included for convenience) is non anomalous. Interestingly, the charges under this non-anomalous U(1) turn out to have the correct $B{-}L$ structure. For instance, fields transforming in the $(\mathbf{3},\mathbf{2},\mathbf{1})$ or $(\mathbf{3},\mathbf{1},\mathbf{2})$ representations have $Q_{\rm diag}$ charge is $-2(1/3-1/2)=1/3$, correct for quark fields. Again, the discussion for the complete spectrum is postponed to the explicit examples in section 3.4.

Notice, as above, that normalization of $B{-}L$ generator is N dependent and leads to $k_{B-L} = 8/3 + 8(N-2)$. Since hypercharge is given by $Y = -T_R^3 + Q_{B-L}$ (with T_R^3 the diagonal generator of $\mathrm{SU}(2)_R$) the values (3.3) and, thus, (3.4) are reobtained for hypercharge normalization and Weinberg angle.

We find it is quite remarkable that the seemingly complicated hypercharge structure in the standard model is easily accomplished by the structure of the diagonal U(1) in this class of orbifold models.

We conclude by remarking that in cases with additional non-anomalous U(1)'s Q_c (2.21), they could be used as hypercharge or $B{-}L$ generators, as long as the U(1) factors in U(3) and U(2) belong to the corresponding linear combination in Q_c (as in the $\mathbb{Z}_6$ example in section 2.3). However, since the presence of these U(1)'s is not generic, we will not analyze this possibility in detail. Moreover, they are not present in the case of $\mathbb{Z}_3$ singularity, which is the only candidate to produce three-generation models.

3.2 Generalities for $\mathbb{C}^3/\mathbb{Z}_3$

In the following we construct some explicit examples of standard model or Left-Right symmetric theories based on the $\mathbb{C}^3/\mathbb{Z}_3$ singularity. This is the most attractive case, since it leads naturally to three-family models. It also illustrates the general technology involved in model building using branes at singularities.

Consider a set of D3-branes and D7_r-branes at a $\mathbb{C}^3/\mathbb{Z}_3$ orbifold singularity, generated by the twist $v=\frac{1}{3}(1,1,-2)$. Its action on the Chan-Paton factors is in general given by the matrices

$$\begin{aligned}\gamma_{\theta,3} &= \mathrm{diag}\left(\mathrm{I}_{n_0},\alpha\mathrm{I}_{n_2},\alpha^2\mathrm{I}_{n_3}\right); \qquad \gamma_{\theta,7_1} = -\,\mathrm{diag}\left(\mathrm{I}_{u_0^1},\alpha\mathrm{I}_{u_2^1},\alpha^2\mathrm{I}_{u_3^1}\right),\\ \gamma_{\theta,7_3} &= \mathrm{diag}\left(\mathrm{I}_{u_0^3},\alpha\mathrm{I}_{u_1^3},\alpha^2\mathrm{I}_{u_2^3}\right); \qquad \gamma_{\theta,7_2} = -\,\mathrm{diag}\left(\mathrm{I}_{u_0^2},\alpha\mathrm{I}_{u_2^2},\alpha^2\mathrm{I}_{u_3^2}\right)\end{aligned} \tag{3.6}$$

with $\alpha=e^{2\pi i/3}$. The notation, slightly different from that in section 2.1, is more convenient for $\mathbb{C}^3/\mathbb{Z}_3$.

The full spectrum is given by

$$\begin{array}{rl}\mathbf{33} & \mathrm{U}(n_0)\times\mathrm{U}(n_1)\times\mathrm{U}(n_2)\\ & 3\left[(n_0,\overline{n}_1)+(n_1,\overline{n}_2)+(n_2,\overline{n}_0)\right],\\ \mathbf{37_r},\mathbf{7_r3} & (n_0,\overline{u}^r{}_1)+(n_1,\overline{u}^r{}_2)+(n_2,\overline{u}^r{}_0)+(u_0^r,\overline{n}_1)+(u_1^r,\overline{n}_2)+(u_2^r,\overline{n}_0)\,.\end{array} \tag{3.7}$$

The superpotential[5] terms are

$$W=\sum_{i=0}^{2}\sum_{r,s,t=1}^{3}\epsilon_{rst}\,\mathrm{Tr}\left(\Phi^r_{i,i+1}\Phi^s_{i+1,i+2}\Phi^t_{i+2,i}\right)+\sum_{i=0}^{2}\sum_{r=1}^{3}\mathrm{Tr}\left(\Phi^r_{i,i+1}\Phi^{37_r}_{i+1,i+2}\Phi^{7_r3}_{i+2,i}\right). \tag{3.8}$$

The twisted tadpole cancellation conditions are

$$\mathrm{Tr}\,\gamma_{\theta,7_3}-\mathrm{Tr}\,\gamma_{\theta,7_1}-\mathrm{Tr}\,\gamma_{\theta,7_2}+3\,\mathrm{Tr}\,\gamma_{\theta,3}=0\,, \tag{3.9}$$

where the relative signs, coming from the sine prefactors, cancel those in the definitions in (3.6), hence it is consistent to ignore both. Equations (3.9) are equivalent to the non-abelian anomaly cancellation conditions.

3.3 Standard model and branes at $\mathbb{C}^3/\mathbb{Z}_3$ singularity

Following the general arguments in section 3.1, the strategy to obtain a field theory with standard model gauge group from the $\mathbb{Z}_3$ singularity is to choose a D3-brane Chan-Paton embedding

$$\gamma_{\theta,3}=\mathrm{diag}\left(\mathrm{I}_3,\alpha\mathrm{I}_2,\alpha^2\mathrm{I}_1\right). \tag{3.10}$$

The simplest way to satisfy the tadpole conditions (3.9) is to introduce only one set of D7-branes, e.g. D7_3-branes, with Chan-Paton embedding $u_0^3=0$, $u_0^1=3$, $u_0^2=6$.

[5] It is possible to consider the generic case of D7^β-branes, with world-volume defined by $\sum_r\beta_rY_r=0$, which preserve the $\mathcal{N}=1$ supersymmetry of the configuration for arbitrary complex β_r [31]. The 37^β, $7^\beta3$ spectra are as above, but the superpotential is $W=\sum_i\sum_r\beta_r\,\mathrm{Tr}(\Phi^r\Phi^{37^\beta}\Phi^{7^\beta3})$, with fields from a single mixed sector coupling to 33 fields from all complex planes.

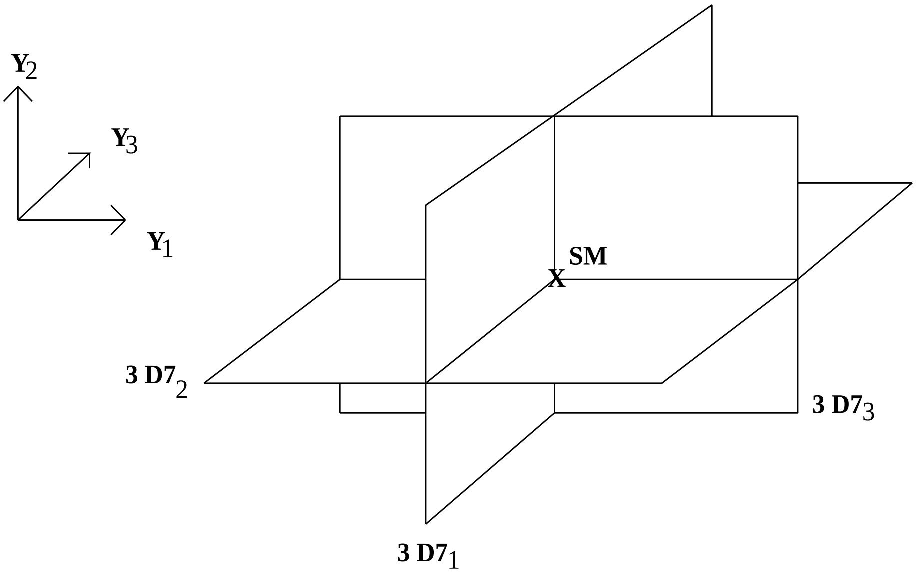

Figure 2: A non-compact type-IIB $\mathbb{Z}_3$ orbifold singularity yielding SM spectrum. Six D3-branes sit on top of a $\mathbb{Z}_3$ singularity at the origin. Tadpoles are canceled by the presence of intersecting D7-branes with their worldvolumes transverse to different complex planes.

Matter fields	Q_3	Q_2	Q_1	$Q_{u_1^r}$	$Q_{u_2^r}$	Y
33 sector						
$3(3,2)$	1	-1	0	0	0	$1/6$
$3(\bar{3},1)$	-1	0	1	0	0	$-2/3$
$3(1,2)$	0	1	-1	0	0	$1/2$
$\mathbf{37}_r$ sector						
$(3,1)$	1	0	0	-1	0	$-1/3$
$(\bar{3},1;2')$	-1	0	0	0	1	$1/3$
$(1,2;2')$	0	1	0	0	-1	$-1/2$
$(1,1;1')$	0	0	-1	1	0	1
$\mathbf{7}_r\mathbf{7}_r$ sector						
$3(1;2)'$	0	0	0	1	-1	0

Table 1: Spectrum of SU(3) × SU(2) × U(1) model. We present the quantum numbers under the $\mathrm{U}(1)^9$ groups. The first three U(1)'s come from the D3-brane sector. The next two come from the $\mathrm{D7}_r$-brane sectors, written as a single column with the understanding that e.g. fields in the $\mathbf{37}_r$ sector are charged under the U(1) in the $\mathbf{7_r 7_r}$ sector.

The gauge group on the D3-branes is $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1)$, whereas in the $\mathrm{D7}_3$-branes is $\mathrm{U}(3) \times \mathrm{U}(6)$ on each. Note that, before compactification, the latter behave as global symmetries in the worldvolume of the D3-branes. The $\mathrm{D7}_3$-branes group can be further broken by global effects, since the corresponding branes are extended along some internal dimensions.

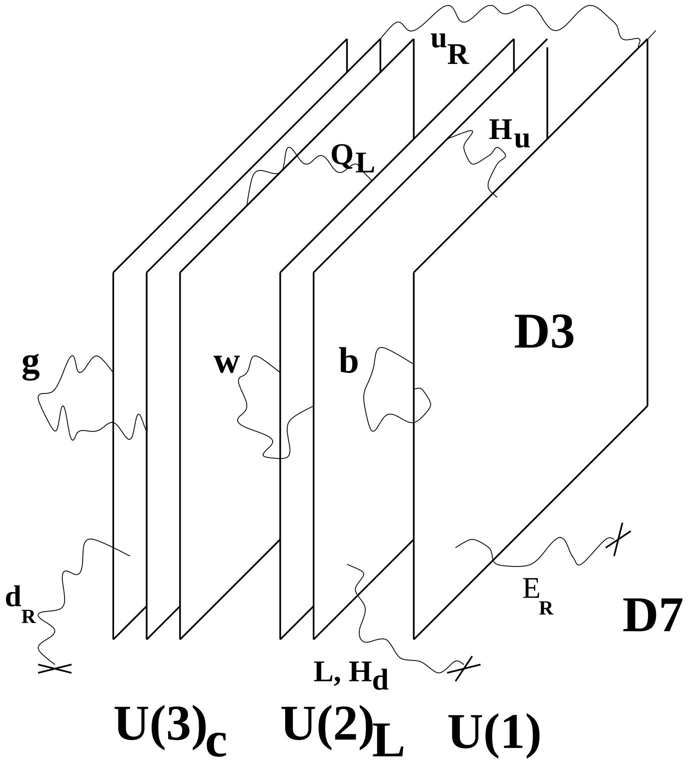

Figure 3: D-brane configuration of a SM $\mathbb{Z}_3$ orbifold model. Six D3-branes (with world-volume spanning Minkowski space) are located on a $\mathbb{Z}_3$ singularity and the symmetry is broken to $U(3) \times U(2) \times U(1)$. For the sake of visualization the D3-branes are depicted at different locations, even though they are in fact on top of each other. Open strings starting and ending on the same sets of D3-branes give rise to gauge bosons; those starting in one set and ending on different sets originate the left-handed quarks, right-handed U-quarks and one set of Higgs fields. Leptons, and right-handed D-quarks correspond to open strings starting on some D3-branes and ending on the D7-branes (with world-volume filling the whole figure).

An alternative procedure to obtain a smaller group on the D7-branes is to use all three kinds of D7-branes, as depicted in figure 2. For instance, a very symmetrical choice consistent with (3.10) is $u_0^r = 0$, $u_1^r = 1$, $u_1^r = 2$, for $r = 1, 2, 3$. Each kind of D7-brane then carries a $U(1) \times U(2)$ group.

The spectrum for this latter model is given in table 1 (for later convenience we have also included states in the $7_r 7_r$ sectors; their computation is analogous to the computation of the 33 sector). In the last column we give the charges under the anomaly-free combination (3.2):

$$Y \; = - \left(\frac{1}{3} Q_3 + \frac{1}{2} Q_2 + Q_1 \right) . \tag{3.11}$$

As promised, it gives the correct hypercharge assignments for standard model fields. A pictorial representation of this type of models is given in figure 3.

We find it remarkable that such a simple configuration produces a spectrum so close to that of the standard model. In particular, we find encouraging the elegant appearance of hypercharge within this framework, as the only linear combination (2.23) of U(1) generators which is naturally free of anomalies in systems of D3-branes at orbifold singularities.

There is another interesting advantage in the fact that hypercharge arises exclusively from the **33** sector. Notice that it allows for the fields in the 77 sector to acquire non-vanishing vev's[6] without breaking hypercharge. These vevs can be used to further break the **77** gauge groups, and produce masses for the extra triplets in the **37** sectors. Then the only remaining light triplets are those of the standard model. Notice also that the same argument does not apply to the doublets in **37** sectors, which remain massless even after the **77** fields acquire vevs. Hence the three families of leptons remain light. This behavior is reminiscent of the models in [14], basically because (see appendix E) their realistic sector has (in a T-dual version) a local structure very similar to our non-compact $\mathbb{Z}_3$ singularity.

The model constructed above, once embedded in a global context, may provide the simplest semirealistic string compactifications ever built. Indeed, in section 4 we will provide explicit compact examples of this kind. Let us once again emphasize that, however, many properties of the resulting theory will be independent of the particular global structure used to achieve the compactification, and can be studied in the non-compact version presented above, as we do in section 5.

3.4 Left-Right symmetric models and the $\mathbb{C}^3/\mathbb{Z}_3$ singularity

One may use a similar approach to construct three-generation models with Left-Right symmetric gauge group. Following the arguments in section 3.1, we consider the D3-brane Chan-Paton embedding

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2\right). \tag{3.12}$$

The corresponding tadpoles can be canceled for instance by D7_r-branes, $r = 1, 2, 3$ with the symmetric choice $u_0^r = 0$, $u_1^r = u_2^r = 1$. The gauge group on D3-branes is $\mathrm{U}(3) \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R$, while each set of D$7_r$-branes contains $\mathrm{U}(1)^2$. As explained above, the combination (3.5)

$$Q_{B-L} = -2\left(\frac{1}{3}Q_3 + \frac{1}{2}Q_L + \frac{1}{2}Q_R\right) \tag{3.13}$$

is non anomalous, and in fact behaves as $B-L$. The spectrum for this model, with the relevant U(1) quantum numbers is given in table 2. A pictorial representation of this type of models is given in figure 4.

[6]Strictly speaking, **77** fields are not dynamical from the point of view of the D3-brane field theory in the non-compact context. This comment should be regarded as applied to compact models with a local behaviour given by the above system of D3- and D7_r-branes at a $\mathbb{Z}_3$ singularity, like those in section 5.

Matter fields	Q_3	Q_L	Q_R	$Q_{U_1^i}$	$Q_{U_2^i}$	$B-L$
33 sector						
$3(3,2,1)$	1	-1	0	0	0	$1/3$
$3(\bar{3},1,2)$	-1	0	1	0	0	$-1/3$
$3(1,2,2)$	0	1	-1	0	0	0
$\mathbf{37}_r$ sector						
$(3,1,1)$	1	0	0	-1	0	$-2/3$
$(\bar{3},1,1)$	-1	0	0	0	1	$2/3$
$(1,2,1)$	0	1	0	0	-1	-1
$(1,1,2)$	0	0	-1	1	0	1
$\mathbf{7}_r\mathbf{7}_r$ sector						
$3(1)'$	0	0	0	1	-1	0

Table 2: Spectrum of $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R$ model. We present the quantum numbers under the $\mathrm{U}(1)^9$ groups. The first three U(1)'s arise from the D3-brane sector. The next two come from the $\mathrm{D}7_r$-brane sectors, and are written as a single column with the understanding that $\mathbf{37}_r$ fields are charged under U(1) factors in the $\mathbf{7_r7_r}$ sector.

We can see that the triplets from the 37_r sectors can become massive after the singlets of the $\mathbf{7_r7_r}$ sector acquire a non-vanishing vev, leaving a light spectrum really close to Left-Right theories considered in phenomenological model-building.

Left-Right symmetric models have several interesting properties that were recently emphasized in [16], (besides the original discussions in [32]). As a practical advantage, it is the simplest to construct, and it allows to distinguish easily the Higgs fields from the leptons, since they transform differently under the gauge group. Phenomenologically, it allows to have gauge unification at the intermediate scale. Most of the properties studied in [16], regarding gauge unification, proton stability, fermion masses, etc. are inherited by theories based on the above construction. We will leave the phenomenological discussion to section 5.

We conclude this section with a comment that applies to models both of SM and LR type. A familiar property of D3-branes at singularities is that sets of D3-branes with traceless Chan-Paton embedding can combine and move off the singularity into the bulk. From the viewpoint of the world-volume field theory this appears as a flat direction along which the gauge group is partially broken, and which is parametrized by a modulus associated to the bulk position of the set of branes. This process is possible in the models discussed above, due to the existence of sets of D3-branes with Chan-Paton embedding $\mathrm{diag}(1,\alpha,\alpha^2)$ (one such set for SM type configurations, and two for LR type models). In fact, it is easy to find the corresponding flat directions in the field theory we computed. This fact will be important in the discussion of some compactified models in section 5.

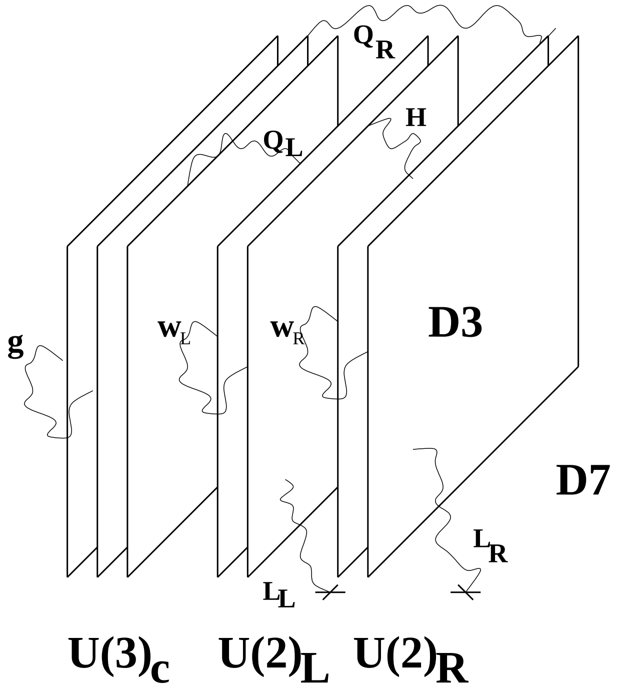

Figure 4: D-brane configuration of a LR $\mathbb{Z}_3$ orbifold model. Seven D3-branes (with worldvolume spanning Minkowski space) are located on a $\mathbb{Z}_3$ singularity and the symmetry is broken to $U(3) \times U(2) \times U(2)$. For the sake of visualization the D3-branes are depicted at different locations, even though they are actually coincident. Open strings starting and ending on the same sets of D3-branes give rise to gauge bosons; those starting and ending on different sets originate the quarks and Higgs fields. Leptons correspond to open strings starting on some D3-branes and ending on D7-branes.

3.5 Non-supersymmetric singularities

In the supersymmetric case the $\mathbb{C}^3/\mathbb{Z}_3$ singularity is singled out. It is the only case leading to three-generation models, due to the fact that three complex directions are equally twisted. The situation is quite different for non-supersymmetric singularities. In fact, by looking at the **33** fermionic spectrum (2.8), namely $\sum_{\alpha=1}^{4}\sum_{i=0}^{N-1}(n_i, \overline{n}_{i+a_\alpha})$, we notice that for $a_1 = a_2 = a_3 = a$ (then $a_4 = -3a$) we have

$$3(n_i, \overline{n}_{i+a}) + (n_i, \overline{n}_{i-3a}) \tag{3.14}$$

and therefore a potential triplication. This singularities are therefore well suited for model building of non-supersymmetric realistic spectra. We would like to point out that, despite the lack of supersymmetry, these models do not contain tachyons, neither in open nor in closed (untwisted or twisted) string sectors.

Without loss of generality we can choose $a = 1$[7] corresponding to $\mathbb{Z}_N$ twist given

[7] For a $\mathbb{Z}_N$ singularity, we must have $\gcd(a, N) = 1$, hence there exists p such that $pa = 1 \bmod N$, and we may choose θ^p to generate $\mathbb{Z}_N$. This only implies a harmless redefinition on the n_i's in $\gamma_{\theta,3}$.

by $(1,1,1,-3)/N$. Thus, we observe that $N=2$ leads to non-chiral theories, $N=4$ to four-generation models, while each model with $N\geq 5$ leads to three-generations models. For instance, by choosing $n_0=3$, $n_1=2$ and $n_i=1$ for $i=3,\ldots,N-1$, a Standard Model gauge group $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y\times\mathrm{U}(1)^{N-1}$ is found with **33** fermions transforming as

$$3\left[(\mathbf{3},\mathbf{2})_{1/6}+(\overline{\mathbf{3}},\mathbf{1})_{-2/3}+(\mathbf{1},\mathbf{2})_{1/2}\right]+(N-3)\text{ singlets}\,. \tag{3.15}$$

Thus, we obtain three generations of left-handed quarks and right-handed U type quarks. Since such a matter content is anomalous, extra contributions coming from D7-branes are expected to complete the spectrum to the three generations of SM quarks and leptons. Notice that presence of D7-branes only produce fundamentals of **33** groups in **37** + **73** sectors and therefore the number of generations is not altered.

Interestingly enough, the correct SM hypercharge assignments above correspond to the anomaly-free diagonal U(1) combination (2.23)

$$Y=-Q_{\mathrm{diag}}=-\left(\frac{1}{3}Q_{n_0}+\frac{1}{2}Q_{n_1}+\sum_{j=3}^{N-1}Q_{n_j}\right). \tag{3.16}$$

Hence hypercharge can arise by the same mechanism discussed in section 3.1 for supersymmetric models.

It is interesting to study the Weinberg angle prediction for these models. The computation follows that in section 3.1, leading to the result (3.4), which gives $\sin^2\theta_W=0.214, 0.115,\ldots$ for $N=3, N=5,\ldots$, respectively. Hence we see that, even though many non-supersymmetric singularities lead to three-generation models, in general they yield too low values of the Weinberg angle. It is interesting that the value gets worse as the order of the singularity increases, suggesting that simple configurations are better suited to reproduce realistic particle models.

A $\mathbb{Z}_5$ example. As a concrete example of the above discussion let us consider the $\mathbb{Z}_5$ singularity acting on the **4** with twist $(a_1,a_2,a_3,a_4)=(1,1,1,-3)$. Hence the action on the **6** is given by $(b_1,b_2,b_3)=(2,2,2)$. The general D3-brane Chan-Paton matrix has the form

$$\gamma_{\theta,3}=\mathrm{diag}\left(\mathrm{I}_{n_0},\alpha\mathrm{I}_{n_1},\alpha^2\mathrm{I}_{n_2},\alpha^3\mathrm{I}_{n_3},\alpha^4\mathrm{I}_{n_4}\right) \tag{3.17}$$

with $\alpha=e^{2\pi i/5}$. Anomaly/tadpole cancellation conditions require

$$4(\alpha-\alpha^4)\,\mathrm{Tr}\,\gamma_{\theta,3}+\sum_{r=1}^{3}\gamma_{\theta,7_r}=0 \tag{3.18}$$

and, therefore, D7-branes must be added. Let us consider the case with only $\mathrm{D7}_3$-branes, with Chan-Paton action

$$\gamma_{\theta,7_3}=\mathrm{diag}\left(\mathrm{I}_{u_0},\alpha\mathrm{I}_{u_1},\alpha^2\mathrm{I}_{u_2},\alpha^3\mathrm{I}_{u_3},\alpha^4\mathrm{I}_{u_4}\right). \tag{3.19}$$

The massless spectrum reads

$$
\begin{array}{lll}
\mathbf{33} & \text{Vectors} & \mathrm{U}(n_0)\times \mathrm{U}(n_1)\times \mathrm{U}(n_2)\times \mathrm{U}(n_3)\times \mathrm{U}(n_4)\,, \\
 & \text{Fermions} & 3\Big[(n_0,\overline{n}_1)+(n_1,\overline{n}_2)+(n_2,\overline{n}_3)+(n_3,\overline{n}_4)+(n_4,\overline{n}_0)] + \\
 & & \quad +(n_0,\overline{n}_2)+(n_1,\overline{n}_3)+(n_2,\overline{n}_4)+(n_3,\overline{n}_0)+(n_4,\overline{n}_1)\Big]\,, \\
 & \text{Cmplx.Sc.} & 3\Big[(n_0,\overline{n}_3)+(n_1,\overline{n}_4)+(n_2,\overline{n}_0)+(n_3,\overline{n}_1)+(n_4,\overline{n}_2)\Big]\,, \\
\mathbf{37_3} & \text{Fermions} & (n_0,\overline{u}_1)+(n_1,\overline{u}_2)+(n_2,\overline{u}_3)+(n_3,\overline{u}_4)+(n_4,\overline{u}_0)\,, \\
 & \text{Cmplx.Sc.} & (n_0,\overline{u}_3)+(n_1,\overline{u}_4)+(n_2,\overline{u}_0)+(n_3,\overline{u}_1)+(n_4,\overline{u}_2)\,, \\
\mathbf{7_33} & \text{Fermions} & (u_0,\overline{n}_1)+(u_1,\overline{n}_2)+(u_2,\overline{n}_3)+(u_3,\overline{n}_4)+(u_4,\overline{n}_0)\,, \\
 & \text{Cmplx.Sc.} & (u_0,\overline{n}_3)+(u_1,\overline{n}_4)+(u_2,\overline{n}_0)+(u_3,\overline{n}_1)+(u_4,\overline{n}_2)\,,
\end{array}
$$

where n_i, u_i's are constrained by (3.18).

To be more specific, let us build up a Left-Right model, by choosing $n_0 = 3$, $n_1 = n_4 = 2$ and $n_2 = n_3 = 1$, which leads to a gauge group $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)_{B-L}\times[\mathrm{U}(1)^5]$. The choice $u_0 = 0$, $u_1 = 3$, $u_2 = u_3 = 7$, $u_4 = 3$ ensures tadpole cancellation. The $B-L$ charge is provided by the anomaly-free diagonal combination (2.23). From the generic spectrum above we find **33** fermions transform, under LR group as

$$
\begin{array}{l}
3\left[(3,2,1)_{1/3}+(1,2,1)_1+(1,1,1)_0+(1,1,2)_{-1}+(\overline{3},1,2)_{-1/3}\right] \\
(3,1,1)_{4/3}+(\overline{3},1,1)_{-4/3}+(1,2,1)_{+1}+(1,1,2)_{-1}+(1,2,2)_0
\end{array}
\tag{3.20}
$$

while scalars live in

$$
3\left[(3,1,1)_{4/3}+(\overline{3},1,1)_{-4/3}+(1,2,1)_{-1}+(1,1,2)_{-1}+(1,2,2)_0\right].
$$

Fermions in **37** **+73** sectors transform as

$$
\begin{array}{l}
(3,1,1;\overline{3},1,1,1)_{-2/3}+(\overline{3},1,1;1,1,1,3)_{2/3}+(1,2,1;1,\overline{7},1,1)_{-1}+ \\
\quad +(1,1,2;1,1,7,1)_1+\text{LR singlets}
\end{array}
\tag{3.21}
$$

while scalars do as

$$
\begin{array}{l}
(3,1,1;1,1,\overline{7},1)_{-2/3}+(1,2,1;1,1,1,\overline{3})_{-1}+(1,1,2;1,\overline{7},1,1)_{-1}+ \\
\quad +(1,1,2;3,1,1,1)_1+(\overline{3},1,1;1,7,1,1)_{2/3}+(1,2,1;1,1,7,1)_1+\text{LR singlets}\,.
\end{array}
\tag{3.22}
$$

We obtain three quark-lepton families from the 33 sector. The remaining fields, vector-like with respect to the LR group could acquire masses by breaking of the 77 groups.

3.6 Other possibilities

It is interesting to compare the kind of models we have constructed, using abelian orbifold singularities, with the field theories arising from D3-branes at more complicated singularities.[8] In this section we discuss some relevant cases. The details for the construction of the corresponding field theories can be found in appendices A, B, C and D.

3.6.1 Orbifold singularities with discrete torsion

Orbifold singularities $\mathbb{C}^3/(\mathbb{Z}_{M_1}\times\mathbb{Z}_{M_2})$ lead to different models depending on their discrete torsion. In appendix A we review the field theory on singularities $\mathbb{C}^3/(\mathbb{Z}_N\times\mathbb{Z}_M\times\mathbb{Z}_M)$, with $\mathbb{Z}_N$ twist $(a_1,a_2,a_3)/N$, and discrete torsion $e^{2\pi i/M}$ between the $\mathbb{Z}_M$ twists. The final spectrum on the D3-brane world-volume is identical to that of the $\mathbb{C}^3/\mathbb{Z}_N$ singularity (2.13), but the superpotential is modified to (A.5).

Hence it follows that the phenomenologically most interesting models in this class are those obtained from the $\mathbb{Z}_3$ orbifold by a further $\mathbb{Z}_M\times\mathbb{Z}_M$ projection with discrete torsion. The resulting spectrum coincides with (3.7), and can lead to three-generation SM or LR models. The superpotential in the 33 sector is modified to

$$W=\mathrm{Tr}\left[\Phi^1_{i,i+1}\Phi^2_{i+1,i+2}\Phi^3_{i+2,i}\right]-e^{2\pi i\frac{1}{M}}\,\mathrm{Tr}\left[\Phi^1_{i,i+1}\Phi^3_{i+1,i+2}\Phi^2_{i+2,i}\right]. \tag{3.23}$$

Since the spectra we obtain from such singularities are identical to those in the simpler case of $\mathbb{C}^3/\mathbb{Z}_3$, we may question the interest of these models. They have two possible applications we would like to mention. The first, explored in section 5, is that discrete torsion enters as a new parameter that modifies the superpotential of the theory, and therefore the pattern of Yukawa couplings in phenomenological models. The second application is related to the process of moving branes off the singularity into the bulk, a phenomenon that, as discussed in section 3 can take place in the realistic models there constructed. In $\mathbb{Z}_N\times\mathbb{Z}_M\times\mathbb{Z}_M$ singularities with discrete torsion, the process can also occur, but the minimum number of branes allowed to move into the bulk is NM^2. In particular, in SM theories constructed from the $\mathbb{C}^3/(\mathbb{Z}_3\times\mathbb{Z}_M\times\mathbb{Z}_M)$ singularity, there are only six D3-branes with traceless Chan-Paton embedding, so motion into the bulk is forbidden for $M\geq 2$. For LR models, motion into the bulk of the twelve traceless D3-branes is forbidden for $M\geq 3$. In fact, the trapping is only partial, since branes are still allowed to move along planes fixed under some $\mathbb{Z}_M$ twist [33]. This partial trapping will be exploited in section 4.1 in the construction of certain compact models.

[8] We have not discussed $\mathbb{C}^3/(\mathbb{Z}_N\times\mathbb{Z}_M)$ singularities. However, it is a simple exercise to check they lead to models similar to those of $\mathbb{C}^3/\mathbb{Z}_N$ singularities. In particular, realistic SM and LR gauge groups are easily achieved by choosing suitable Chan-Paton actions, and hypercharge (or the $B-L$ U(1) in LR models) arises as a diagonal combination, straightforward generalization of (2.23). However, no three-generation models exist within this class. Also, the Weinberg angle, given by (3.4) by replacing N by NM, is too small for any singularity of this type.

3.6.2 Non-abelian orbifold singularities

We may also consider the field theories on D3-branes at non-abelian orbifold singularities. The rules to compute the spectrum, along with the relevant notation, are reviewed in appendix B. They allow to search for phenomenologically interesting spectra. The classification of non-abelian discrete subgroups of SU(3) and several aspects of the resulting field theories have been explored in [34, 35]. An important feature in trying to embed the standard model in such field theories is that of triplication of families. Seemingly there is no non-abelian singularity where the spectrum appears in three identical copies. A milder requirement with a chance of leading to phenomenological models would be the appearance of three copies of at least one representation, i.e. $a_{ij}^{\mathbf{3}} = 3$ for suitable $i \neq j$. Going through the explicit tables in [34] there is one group with this property, Δ_{3n^2} for $n = 3$, on which we center in what follows. The 33 spectrum[9] for this case is

$$\prod_{i=1}^{9} \mathrm{U}(n_i) \times \mathrm{U}(n_{10}) \times \mathrm{U}(n_{11})$$
$$\sum_{i=1}^{9} (n_i, \overline{n}_{10}) + 3\,(n_{10}, \overline{n}_{11}) + \sum_{i=1}^{9} (n_{11}, \overline{n}_i)\,. \tag{3.24}$$

If the triplicated representation is chosen to give left-handed quarks, a potentially interesting choice is given by $n_i = 1$, $n_{10} = 3$, $n_{11} = 2$. However, and due to the additional factors r_i (in our case $r_i = 1$ for $i = 1, \ldots, 9$, $r_{10} = r_{11} = 3$) in (B.4), the diagonal combination does not lead to correct hypercharge assignments. Besides (B.4) there are eight non-anomalous U(1)'s given by combinations $Q_c = \sum_{i=1}^{9} c_i Q_{n_i} + Q_{n_{10}} + \frac{3}{2} Q_{n_{11}}$, with $\sum_{i=1}^{9} c_i = 9$. The charge structure under the combination $c_1 = c_2 = c_3 = 3$, $c_i = 0$ for $i = 4, \ldots, 9$ is particularly interesting. We obtain

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \left(\times\, \mathrm{U}(1)^8\right)$$
$$3(3,2)_{1/6} + 3(1,2)_{1/2} + 6(1,2)_{-1/2} + 3(\overline{3},1)_{-2/3} + 6(\overline{3},1)_{1/3}\,. \tag{3.25}$$

Hence, the spectrum under this U(1) contains fields present in the standard model, and with the possibility of leading to three net copies (if suitable 37, 73 sectors are considered), even though they would still be distinguished by their charges under the additional U(1)'s. It is conceivable that this model leads to phenomenologically interesting field theories by breaking the additional U(1) symmetries at a large enough scale. However, it is easy to see that the Weinberg angle, which, taking into account (B.3), is still given by (3.4) where now $N = 11$, is exceedingly too small $\sin^2\theta_W = 3/62$. Nevertheless we hope the model is illustrative on the type of field theory spectra one can achieve using non-abelian singularities.

[9] The gauge group in [34, page 16] corresponds to the particular case of Chan-Paton embedding given by the regular representation. Here we consider a general choice.

3.6.3 Non-orbifold singularities

In appendix C we discuss the construction of field theories on D3-branes at some non-orbifold singularities, which is in general rather involved. We also discuss that a promising spectrum is obtained from a partial blow-up of a $\mathbb{Z}_3$ quotient of the conifold. The general spectrum is given in (C.7) and (C.9). There are several possibilities to construct phenomenologically interesting spectra from this field theory. For the purpose of illustration, let us consider one example with $n_2 = 3$, $n_0 = 2$, $n_1' = 2$, $n_1' = 1$ and $v_2 = 1$, $x_2 = 3$, $x_3 = 5$ (all others vanishing). This choice leads to the spectrum

$$\begin{array}{rl} & \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{\mathrm{diag}} \times \mathrm{U}(1)' \\ \mathbf{33} & 3(3,2,1)_{1/3} + (\overline{3},1,1)_{-4/3} + 2(\overline{3},1,2)_{-1/3} + (1,2,2)_0 + 2(1,2,1)_1 + (1,1,2)_{-1} \\ \mathbf{37},\mathbf{73} & (3,1,1;1,1)_{-2/3} + (1,1,1;1,1)_2 + (3,1,1;\overline{3},1)_{-2/3} + (1,1,2;3,1)_1 + \\ & + (1,2,1;1,\overline{5})_{-1} + (\overline{3},1,1;1,5)_{2/3} \end{array} \tag{3.26}$$

with subindices giving charges under $\mathrm{U}(1)_{\mathrm{diag}}$. We obtain three net generations, but the right-handed quarks have a different embedding into the Left-Right symmetry: whereas two generations of right-handed quarks are standard $\mathrm{SU}(2)_R$ doublets the other generation are $\mathrm{SU}(2)_R$ singlets. Notice that an interesting property this model illustrates is that one can achieve three quark-lepton generations without triplication of the $(1,2,2)$ Higgs multiplets. This could be useful in order to suppress flavour changing neutral currents for these models.

3.6.4 Orientifold singularities

There is another kind of singularities that arises naturally in string theory, which we refer to as orientifold singularities. They arise from usual geometric singularities which are also fixed under an action Ωg, where Ω reverses the world-sheet orientation, and g is an order two geometric action. Orientifolds were initially considered in [36] and have recently received further attention [37, 38, 39].

Starting with the field theory on a set of D3-branes at a usual singularity, the main effect of the orientifold projection is to impose a $\mathbb{Z}_2$ identification on the fields. Most examples in the literature deal with orientifolds of orbifolds singularities [40, 24], on which we center in the following (see [41] for orientifold of some simple non-orbifols singularities). To be concrete, we consider orientifolds of $\mathbb{C}^3/\mathbb{Z}_N$ singularities with odd N [24]. The spectrum before the orientifold projection is given in (2.13). This configuration can be modded out by $\Omega R_1 R_2 R_3 (-1)^{F_L}$ (where R_r acts as $Y_r \to -Y_r$, and F_L is left-handed world-sheet fermion number), preserving $\mathcal{N} = 1$ supersymmetry. The action of the orientifold projection on the 33 spectrum amounts to identifying the gauge groups $\mathrm{U}(n_i)$ and $\mathrm{U}(n_{-i})$, in such a way (due to world-sheet orientation reversal) that the fundamental representation n_i is identified with the anti-fundamental $\overline{n}_{-i}$. Consequently, there is an identification of the chiral multiplet $\Phi^r_{i,i+a_r}$ and $\Phi^r_{-i-a_r,-i}$. Finally, since Ω exchanges the open string endpoints, 37_r fields $\Phi^{(37_r)}_{i,i-\frac{1}{2}b_r}$ map to 7_r3 fields $\Phi^{7_r3}_{N-i+\frac{1}{2}b_r,N-i}$.

Notice that the gauge group $\mathrm{U}(n_0)$ is mapped to itself and, similarly, chiral multiplets $\Phi^r_{i,i+a_r}$ are mapped to themselves if $i+a_r = N-i$. There exist two possible orientifold projections, denoted "SO" and "Sp", differing in the prescription of these cases. The SO projection projects the $\mathrm{U}(n_0)$ factor down to $\mathrm{SO}(n_0)$, and the bi-fundamental $\Phi^r_{i,i+a_r} (= \Phi^r_{i,-i})$ to the two-index antisymmetric representation of the final $\mathrm{U}(n_i)$. The Sp projection chooses instead $\mathrm{USp}(n_0)$, and two-index symmetric representations.

It is easy to realize that D3-branes at orientifold singularities will suffer from a generic difficulty in yielding realistic spectra. The problem lies in the fact that the orientifold projection removes from the spectrum the diagonal U(1) (2.23) (as is obvious, since the U(1) in $\mathrm{U}(n_0)$ is automatically lost), which was crucial in obtaining correct hypercharge in our models in section 3.

For illustration, we can check explicitly that, in the only candidate to yield three-family models, the orientifold of the $\mathbb{C}^3/\mathbb{Z}_3$ singularity, no realistic spectra arise. The general 33 spectrum for this orientifold (choosing, say the SO projection) is

$$\begin{aligned}&\mathrm{SO}(n_0)\times\mathrm{U}(n_1)\\&3\left[(n_0,\overline{n}_1)+\left(1,\frac{1}{2}n_1(n_1-1)\right)\right].\end{aligned} \tag{3.27}$$

The U(1) factor is anomalous, with anomaly canceled by a GS mechanism [24].

A seemingly interesting possibility would be $\mathrm{SO}(3)\times\mathrm{U}(3)\simeq\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$. However, the U(1) factor does not provide correct hypercharge assignments. Moreover, it is anomalous and therefore not even present in the low-energy theory. A second possibility, yielding a Pati-Salam model $\mathrm{SO}(4)\times\mathrm{U}(4)\simeq\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{SU}(4)\times\mathrm{U}(1)$ is unfortunately vector-like. In fact, the most realistic spectrum one can construct is obtained for $n_0=1$, $n_1=5$, yielding a U(5) (actually SU(5)) gauge theory with chiral multiplets in three copies of $\overline{\mathbf{5}}+\mathbf{10}$. This GUT-like theory, which constitutes a subsector in a compact model considered in [10], does not however contain Higgs fields to trigger breaking to the standard model group.

We hope this brief discussion suffices to support our general impression that orientifold singularities yield, in general, field theories relatively less promising than orbifold singularities.

3.6.5 Non-supersymmetric models from antibranes

We would like to conclude this section by considering a further set of field theories, obtained by considering branes and antibranes at singularities. The rules to compute the spectrum are reviewed in appendix C. For simplicity we center on systems of branes and antibranes at $\mathbb{C}^3/\mathbb{Z}_N$ singularities, with $\mathbb{Z}_N \subset \mathrm{SU}(3)$, even though the class of models is clearly more general. Notice that, due to the presence of branes and antibranes, the resulting field theories will be non supersymmetric. Let us turn to the discussion of the generic features to be expected in embedding the standard model in this type of D3/$\overline{\mathrm{D3}}$ systems.

The first possibility we would like to consider is the case with the standard model embedded on D3- and $\overline{\mathrm{D3}}$-branes. From (D.1), tachyons arise whenever the Chan-Paton embedding matrices $\gamma_{\theta,3}$, $\gamma_{\theta,\bar{3}}$ have some common eigenvalue. Denoting n_j, m_j the number of eigenvalues $e^{2\pi i j/N}$ in $\gamma_{\theta,3}$, $\gamma_{\theta,\bar{3}}$, tachyons are avoided only if one considers models where n_i vanishes when m_i is non zero, and vice-versa. This corresponds, for instance, to embedding SU(3) in the D3-branes and SU(2) in the $\overline{\mathrm{D3}}$-branes.

One immediate difficulty is manifest already at this level, regarding the appearance of hypercharge. As we remark in appendix D, it is a simple exercise to extend the analysis of U(1) anomalies of section 2.3 to the case with antibranes, with the result that (generically) there are two non-anomalous diagonal combinations (2.23) if no n_i, m_i vanish. But this case is excluded by the requirement of absence of tachyons. Hence tachyon-free models lack the diagonal linear combination of U(1)'s and in general fail to produce correct hypercharge. It is still possible that in certain models, suitable tachyon-free choices of n_i, m_i may produce hypercharge out of non-diagonal additional U(1)'s. Instead of exploring this direction (which in any case would not be available in the only three-family case of the $\mathbb{C}^3/\mathbb{Z}_3$ singularity), we turn to a different possibility.

An alternative consists in embedding the standard model in, say D3-branes, but to satisfy the tadpole cancellation conditions using $\overline{\mathrm{D7}}$-branes (and possibly D7-branes as well). The resulting models are closely related to those in section 3, differing from them only in the existence of $3\overline{7}$, $\overline{7}3$ sectors. Let us consider a particular example, with a set of D3-, D7_3- and $\overline{\mathrm{D7}}_3$-branes at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity with twist $v = (1,1,-2)/3$, with Chan-Paton embeddings

$$\gamma_{\theta,3} = \mathrm{diag}(\mathrm{I}_3, \alpha\mathrm{I}_2, \alpha^2\mathrm{I}_1)\,, \qquad \gamma_{\theta,7_3} = \mathrm{diag}(\alpha^2\mathrm{I}_3)\,, \qquad \gamma_{\theta,\overline{7}_3} = \mathrm{diag}(\mathrm{I}_3) \tag{3.28}$$

which satisfy the tadpole condition

$$\mathrm{Tr}\,\gamma_{\theta,7_3} - \mathrm{Tr}\,\gamma_{\theta,\overline{7}_3} + 3\,\mathrm{Tr}\,\gamma_{\theta,3} = 0\,. \tag{3.29}$$

The resulting spectrum on the D3-branes is

$$\begin{array}{ll}
 & \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)_Y \\
\mathbf{33} & 3(3,2)_{1/6} + 3(1,2)_{1/2} + 3(\overline{3},1)_{-2/3} \\
\mathbf{37_3},\,\mathbf{7_33} & (1,2;\overline{3},1)_{-1/2} + (\overline{3},1;3,1)_{1/3} \\
\mathbf{3\overline{7}_3},\,\mathbf{\overline{7}_33} & (1,1;1,3)_1 + (1,2;1,\overline{3})_{1/2} \quad \text{(left-handed fermions)} \\
 & (3,1;1,\overline{3})_{-1/3} + (\overline{3},1;1,3)_{1/3} \quad \text{(complex scalars)}\,.
\end{array} \tag{3.30}$$

In the supersymmetric sectors, 33, 37 and 73, the above representations correspond to chiral multiplets, while in the non-supersymmetric $3\overline{7}$, $\overline{7}3$ the spectrum for fermions and scalars is different. The complete spectrum corresponds to a three-generation model. Notice that models of this type can be obtained from our constructions in section 3 simply by adding an arbitrary number of $\overline{\rm D7}$-branes with traceless Chan-Paton factors, and annihilating fractional D7- and $\overline{\rm D7}$-branes with identical Chan-Paton phase (this corresponds to the condensation of the corresponding tachyons). Since many of the relevant properties of the models in section 3 are inherited by the non-supersymmetric theories with branes and antibranes, we do not pursue their detailed discussion here.

4. Embedding into a compact space

As discussed in the introduction, even though SM gauge interactions propagate only within the D3-branes even in the non-compact setup, gravity remains ten-dimensional. In order to reproduce correct four-dimensional gravity the transverse space must be compact. In this section we present several simple string compactifications which include the local structures studied in section 3, as particular subsectors.

Since these local structures contain D3- and D7-branes, the corresponding RR charges must be cancelled in the compact space. A simple possibility is to cancel these charges by including antibranes (denoted $\overline{\rm D3}$-, $\overline{\rm D7}$-branes) in the configuration. General rules to avoid tachyons and instabilities against brane-antibrane annihilation have been provided in [13] (see [42, 15] for related models), which we exploit in section 4.1 to construct compact models based on toroidal orbifolds. A second possibility is to use orientifold planes. In section 4.2 we provide toroidal orientifolds without $\overline{\rm D3}$-branes, the D3-brane charge being cancelled by orientifold 3-planes (O3-planes). All of the above models contain some kind of antibranes, breaking supersymmetry in a hidden sector lying within the compactification space. It may be possible to find completely supersymmetric embeddings of our local structure by using toroidal orientifolds including O3- and O7-planes, even though we have not found such examples within the class of toroidal abelian orientifolds. This is hardly surprising, since this class is rather restricted. We argue that suitable $\mathcal{N}=1$ supersymmetric compactifications of our local structures exist in more general frameworks, like compactification on curved spaces. In fact, in section 4.3 we present an explicit F-theory compactification of this type.

4.1 Inside a compact type-IIB orbifold

A simple family of Calabi-Yau compactifications with singularities is given by toroidal orbifolds $\mathbb{T}^6/\mathbb{Z}_N$. The twist $\mathbb{Z}_N$ is constrained to act cristalographycally, and a list of the possibilities preserving supersymmetry is given in [43]. Several such orbifolds include $\mathbb{C}^3/\mathbb{Z}_3$ singularities, the simplest being $\mathbb{T}^6/\mathbb{Z}_3$, with 27 singularities arising from fixed points of $\mathbb{Z}_3$. Parameterizing the three two-tori in $\mathbb{T}^6$ by z_r, $r = 1, 2, 3$, with $z_r \simeq z_r + 1 \simeq z_r + e^{2\pi i/3}$, the 27 fixed points are located at (z_1, z_2, z_3) with $z_r = 0$, $\frac{1}{\sqrt{3}}e^{\pi i/6}$, $\frac{1}{\sqrt{3}}e^{-\pi i/6}$. For simplicity, we denote these values by 0, +1, −1 in what follows. We now present several compact versions of the models in section 3, based on this orbifold.

4.1.1 Simple examples

The simplest possibility, in order to embed the local structures of section 3 in a compact orbifold, is to consider models with only one type of D7-brane. We will also consider the addition of Wilson lines, a modification which, being a global rather than a local phenomenon, was not available in the non-compact case. We first construct an example of a Left-Right symmetric model, which illustrates the general technique, and then briefly mention the construction of a model with standard model group.

A Left-Right symmetric model. Consider a compact $\mathbb{T}^6/\mathbb{Z}_3$ orbifold, and locate seven D3-branes with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_3, \alpha\mathrm{I}_2, \alpha^2\mathrm{I}_2\right) \tag{4.1}$$

at the origin. To cancel twisted tadpoles at the origin we will add six $\mathrm{D7}_1$-branes at the origin in the first complex plane, with Chan-Paton embedding

$$\gamma_{\theta,7} = \mathrm{diag}\left(\alpha\mathrm{I}_3, \alpha^2\mathrm{I}_3\right). \tag{4.2}$$

The local structure around the origin is exactly as in the LR models studied in section 3.4, hence the final model will automatically contain a $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ gauge group, with three quark-lepton generations. That one can make this statement at such an early stage in the construction of the model is the main virtue of the bottom-up approach we are presenting. In the following, we simply discuss how to complete the model satisfying the string consistency conditions, and will not bother computing the detailed spectrum from the additional sectors.

The choice (4.2) cancels the twisted tadpoles at the origin. But the worldvolume of these $\mathrm{D7}_1$-branes also overlap with other eight fixed points $(0, n, p)$, where $n, p = 0, \pm 1$, so we will also have to ensure twisted tadpole cancellation at them. A simple option would be to add one single D3-brane with $\gamma_{\theta,3} = 1$ at each. However we consider the following more interesting possibility, which in addition breaks the (too large) $\mathrm{D7}_1$-brane $\mathrm{U}(3) \times \mathrm{U}(3)$ gauge group. Consider adding a Wilson line e.g. along the second complex plane, and acting on seven-branes as

$$\gamma_{W,7} = \mathrm{diag}\left(\mathrm{I}_1, \alpha\mathrm{I}_1, \alpha^2\mathrm{I}_1, \mathrm{I}_1, \alpha\mathrm{I}_1, \alpha^2\mathrm{I}_1\right). \tag{4.3}$$

The $\mathrm{D7}_1$-brane gauge group is broken down to $\mathrm{U}(1)^6$ (some of which will actually be anomalous and therefore not really present). The fixed points $(0, \pm 1, p)$, $p = 0, \pm 1$ feel the presence of the Wilson line [44, 39], hence the $\mathrm{D7}_1$ contribution to twisted

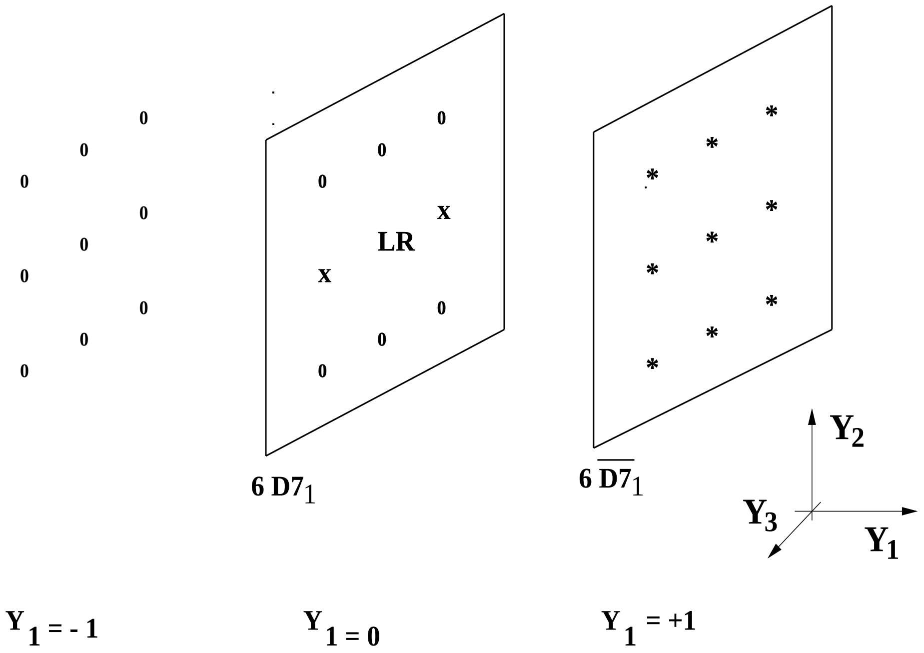

Figure 5: A compact type-IIB $\mathbb{T}^6/\mathbb{Z}_3$ orbifold model with a LR subsector. The points marked (0) correspond to fixed points without D3-branes; those marked ($*$) contain anti-D3-branes and those marked (x) have D3-branes located on them. Seven D3-branes, leading to a LR model of the type studied in section 3.4, reside at the origin. The overall RR charge cancels.

tadpoles is given by $\mathrm{tr}\, \gamma_{\theta,7}\gamma_{W,7}$ or $\mathrm{tr}\, \gamma_{\theta,7}\gamma_{W,7}^2$, which vanish. Hence twisted tadpole cancellation does not require any additional branes at those fixed points. On the other hand the points $(0, 0, \pm 1)$ do not feel the presence of the Wilson line and require e.g. the presence of one D3-brane with $\gamma_{\theta,3} = 1$ on each of them to achieve tadpole cancellation.[10]

The above configuration has a total of nine D3-branes and six D7$_1$-branes. The simplest possibility to cancel untwisted tadpoles is to locate six $\overline{\text{D7}}_1$-branes at a different point, say $Y_1 = 1$, in the first complex dimension, with the same Chan-Paton twist matrix (4.2). Twisted and untwisted tadpoles are cancelled by adding one $\overline{\text{D3}}$-brane at each of the nine fixed points $(1, m, p)$, $m, p = 0, \pm 1$, with Chan-Paton embedding $\gamma_{\theta,\bar{3}} = \mathrm{diag}(\mathrm{I}_1)$. The number of branes minus that of antibranes vanishes. The complete configuration is schematically depicted in figure 5

A comment on the stability of this brane configuration is in order. In this compact configuration the D7-branes and $\overline{\text{D7}}$-branes are trapped at the singular points, which they cannot leave. The flat directions associated to motions into the bulk are not present in their world-volume field theory. Moreover such motions would violate the twisted tadpole cancellation conditions. Hence, these objects cannot leave the

[10] An amusing possibility is to add, instead of these single D3-branes, "new brane-worlds" of seven 3-branes with (4.1), so that the model contains three different but analogous universes.

fixed points and annihilate each other into the vacuum. On the other hand, groups of three D3-branes in the LR subsector may in principle abandon the origin and travel into the bulk in $\mathbb{Z}_3$-invariant configurations, attracted by the $\overline{\mathrm{D3}}$-branes living at the fixed points $(1, m, p)$. Eventually the D3-branes could reach the $\overline{\mathrm{D3}}$-branes, and partial brane-antibrane annihilation would follow. This annihilation cannot be complete, since again the tadpole cancellation conditions would be violated. Since the complete configuration is non supersymmetric, a more detailed analysis of the forces involved would be needed to find out whether the emission of D3-branes from the origin to the bulk is actually dynamically preferred or not, but we expect this specific model to suffer from such instability. On the other hand, as we will argue below, there are other ways to embed the set of SM or LR D3-branes into a compact space in which brane-antibrane annihilation is not possible.

One standard-like model. From the above construction, it is clear how to construct a compact orbifold model with an SM subsector, as we sketch in the following. We place the six D3-branes at the origin with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_3, \alpha\mathrm{I}_2, \alpha^2\mathrm{I}_1\right) \tag{4.4}$$

and nine $\mathrm{D7}_1$-branes with

$$\gamma_{\theta,7} = \mathrm{diag}\left(\alpha\mathrm{I}_3, \alpha^2\mathrm{I}_6\right). \tag{4.5}$$

The local structure at the origin is of the type considered in section 3.3, and will lead to a three-generation $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ subsector. In order to break the $\mathrm{U}(3) \times \mathrm{U}(6)$ $\mathrm{D7}_1$-brane gauge group, we add a Wilson line along the second complex plane, with

$$\gamma_{W,7} = \mathrm{diag}\left(\mathrm{I}_1, \alpha\mathrm{I}_1, \alpha^2\mathrm{I}_1, \mathrm{I}_2, \alpha\mathrm{I}_2, \alpha^2\mathrm{I}_2\right) \tag{4.6}$$

which break the $\mathrm{D7}_1$-brane gauge group to $\mathrm{U}(2)^3 \times \mathrm{U}(1)^3$. Again, the fixed points $(0, \pm1, p)$, $p = 0, \pm1$, feel the Wilson lines and receive no contribution to twisted tadpoles, hence do not require the presence of D3-branes. On the other hand the fixed points $(0, 0, \pm1)$ require additional D3-branes for tadpole cancellation. The simplest option is to add three 3-branes at each of these with $\gamma_{\theta,3} = \mathrm{diag}(\mathrm{I}_2, \alpha\mathrm{I}_1)$.

Let us add nine $\overline{\mathrm{D7}}_1$-branes at e.g. $Y_1 = +1$, with

$$\gamma_{\theta,\overline{7}} = \mathrm{diag}\left(\mathrm{I}_3, \alpha\mathrm{I}_3, \alpha^2\mathrm{I}_3\right). \tag{4.7}$$

We also add a Wilson line W' along the second complex plane, with embedding

$$\gamma_{W',\overline{7}} = \mathrm{diag}\left(\mathrm{I}_3, 1, \alpha, \alpha^2, 1, \alpha, \alpha^2\right). \tag{4.8}$$

Twisted tadpoles are generated only at points of the form $(+1, \pm1, p)$, $p = 0, \pm1$. They can be cancelled by adding two $\overline{\mathrm{D3}}$-branes with $\gamma_{\theta,\overline{3}} = \mathrm{diag}(\alpha, \alpha^2)$ at each. This completes the model, which satisfies the requirements of twisted and untwisted tadpole cancellation.

4.1.2 Trapping of branes through discrete torsion

As noted in section 3.6, orbifold models with discrete torsion may lead to partial trapping of D3-branes along certain directions. In this section we present a model with D3-branes and $\overline{\rm D3}$-branes where this partial trapping is used to avoid brane-antibrane annihilation.

Consider the compact orbifold $T^6/(\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2)$, with discrete torsion in the $\mathbb{Z}_2 \times \mathbb{Z}_2$ factor. We place a set of twelve D3-branes at the origin $(0,0,0)$, with Chan-Paton embedding[11]

$$\gamma_{\theta,3} = \mathrm{diag}\,(\mathrm{I}_6, \alpha \mathrm{I}_4, \alpha \mathrm{I}_2)\,. \tag{4.9}$$

As discussed in appendix A, the additional $\mathbb{Z}_2 \times \mathbb{Z}_2$ twist breaks the gauge group to $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(1)$. In order to ensure cancellation of twisted tadpoles at the origin, we introduce $\mathrm{D7}_1$-branes at $Y_1 = 0$, with Chan-Paton embedding

$$\gamma_{\theta,7_1} = \mathrm{diag}\left(\alpha \mathrm{I}_6, \alpha^2 \mathrm{I}_{12}\right). \tag{4.10}$$

The local structure around the origin is precisely that of the SM theories in section 3.6.1, hence the final model will contain a three-generation $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ sector.

Due to the $\mathbb{Z}_2 \times \mathbb{Z}_2$ twist, the $\mathrm{D7}_1$-brane gauge group would be $\mathrm{U}(3) \times \mathrm{U}(6)$. To reduce it further, we introduce a Wilson line W_2 along the second complex plane, embedded as

$$\gamma_{W_2,7} = \mathrm{diag}\left(\mathrm{I}_2, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2; \mathrm{I}_4, \alpha \mathrm{I}_4, \alpha^2 \mathrm{I}_4\right). \tag{4.11}$$

The surviving $\mathrm{D7}_1$-brane gauge group is $\mathrm{U}(1)^3 \times \mathrm{U}(2)^3$. The $\mathrm{D7}_1$-brane contribution to twisted tadpoles at fixed points $(0, \pm 1, p)$ for $p = 0, \pm 1$, vanishes, hence no D3-branes are required at them. At fixed points of the form $(0,0,\pm 1)$, tadpoles generated by the $\mathrm{D7}_1$-branes can be cancelled by placing D3-branes with $\gamma_{\theta,3} = \mathrm{diag}(\mathrm{I}_4, \alpha \mathrm{I}_2)$. Notice that these D3-branes are stuck at the fixed points.

So far we have placed twelve D3-branes at $(0,0,0)$ and six D3-branes at each of the points $(0,0,\pm 1)$. Also, there are eighteen $\mathrm{D7}_1$-branes at $Y_1 = 0$. In order to cancel the $\mathrm{D7}_1$-brane untwisted tadpole, we introduce nine anti-$\mathrm{D7}_1$-branes at $Y_1 = \pm 1$, with

$$\gamma_{\theta,\overline{7}} = \mathrm{diag}\left(\mathrm{I}_3, \alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_3\right). \tag{4.12}$$

We also add a Wilson line W' along the second complex plane, with embedding

$$\gamma_{W_2',\overline{7}} = \mathrm{diag}\left(\mathrm{I}_3, 1, \alpha, \alpha^2, 1, \alpha, \alpha^2\right). \tag{4.13}$$

They induce no tadpoles at the points of the form $(+1, 0, p)$, $p = 0, \pm 1$. The non-vanishing tadpoles at the points of the form $(+1, \pm 1, p)$, $p = 0, \pm 1$, can be cancelled by adding two anti-D3-branes at each, with $\gamma_{\theta,\overline{3}} = \mathrm{diag}(\alpha, \alpha^2)$. The $\overline{\mathrm{D7}}_1$-branes

[11] Actually, one should specify the embeddings of the $\mathbb{Z}_2$ twists. They are given, up to phases, by the choice (A.2), and we do not give them explicitly to simplify the discussion.

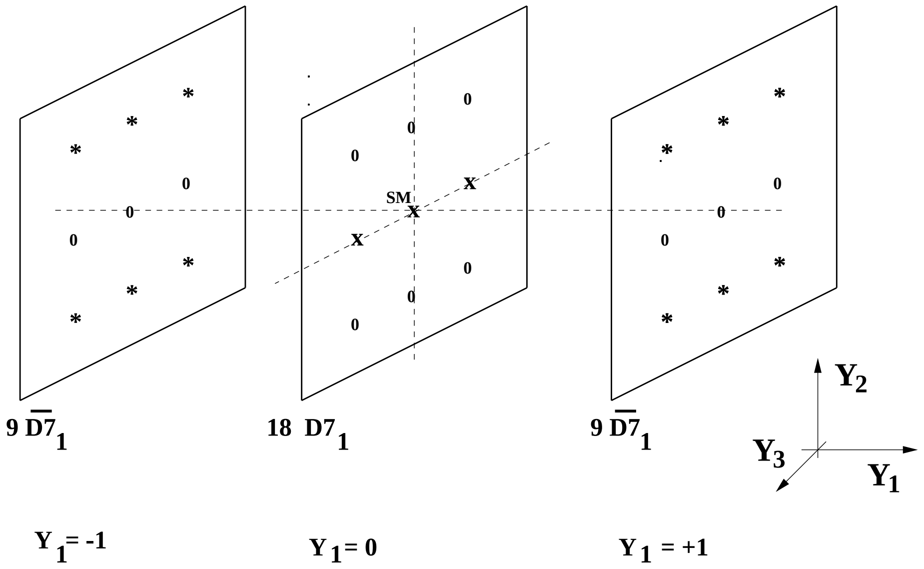

Figure 6: A compact $\mathbb{Z}_3 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold model with discrete torsion. Twelve D3-branes reside at the origin giving rise to the SM. In addition there are six D3-branes at the points marked (x) and two anti-D3-branes at those marked $(*)$. The D3-branes in the SM cannot travel to the bulk and annihilate the anti-D3-branes because they can only move along the straight lines shown.

at $(-1, n, p)$, with $n, p = 0, \pm 1$, and the corresponding $\overline{\mathrm{D3}}$-branes are mapped to the above by the $\mathbb{Z}_2 \times \mathbb{Z}_2$ operations, hence their embedding is determined by the above. The final configuration contains 18 $\mathrm{D7}_1$-branes and 18 $\overline{\mathrm{D7}}_1$-branes, as well as 24 D3-branes and 24 $\overline{\mathrm{D3}}$-branes, and is shown in figure 6. All RR charges are suitably cancelled.

One important virtue of this model (and others of its kind) is that all $\overline{\mathrm{D3}}$-branes are stuck at fixed points, and so are most D3-branes. In fact, the only D3-branes not completely trapped are those in the SM sector at the origin $(0, 0, 0)$. As explained in section 3.6.1, they are however partially trapped, being able to move away at most in only one complex direction. Hence, they can never reach the points $(\pm 1, \pm 1, p)$ where anti-D3-branes sit, and brane-antibrane annihilation is therefore not possible. This example illustrates how to employ partial trapping by discrete torsion to improve the stability properties of the models.

4.2 Inside a compact type-IIB orientifold

In this section we will present an explicit type-IIB orientifold model with the following properties:

1. The standard model is embedded on a D3-brane at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity.

2. There are no anti-D3-branes that could destabilize the system.

3. There are equal number of D7-branes and anti-D7-branes, breaking supersymmetry, trapped at different $\mathbb{Z}_3$ fixed points, providing a clean example of gravity mediated supersymmetry breaking.

Models of this type are similar to the ones constructed in [14, 16] (based on the general framework in [13], see also [15]), but in those cases the standard model was inside a higher-dimensional brane (D7 or D9-brane) and supersymmetry was broken by the presence of lower-dimensional anti-branes ($\overline{\text{D3}}$- or $\overline{\text{D5}}$-branes). Furthermore those examples did not include models where all the anti-branes were completely trapped, and it was left as a dynamical question if the configuration was stable. The class of models we are going to present succeeds in trapping all antibranes. Nevertheless, it is well known that T-dualizing these models with respect to the different extra dimensions we should get to models where the standard model is embedded into D7- or D9-branes and therefore models like those considered in [14, 16]. We will then find the T-duals of these models which happen to be complicated versions of the models in [14, 16] in terms of several Wilson lines. We will submit the reader to the appendices where we discuss in detail the T-duality between the models of this section and those of [14].

4.2.1 Explicit models

As an illustration to this class of models let us consider in detail the following orientifold of the $\mathbb{T}^6/\mathbb{Z}_3$ orbifold. The orientifold action is $\Omega(-1)^{F_L}\, R_1\, R_2\, R_3$ with R_i reflection on the i-th plane. There are 64 orientifold three-planes (O3-planes), which are localized at points in the internal space. To cancel their RR charge we need a total of 32 D3-branes. We will distribute them among the 27 orbifold fixed points (m,n,p), $m,n,p = 0,\pm 1$. We will also introduce an equal number of D7- and $\overline{\text{D7}}$-branes. Among these 27 points, only the origin $(0,0,0)$ is fixed under the orientifold action, hence it is an orientifold singularity, of the type mentioned in section 3.6.4. The cancellation of tadpoles at this point requires

$$3\,\mathrm{Tr}\,\gamma_{\theta,3} + \left(\mathrm{Tr}\,\gamma_{\theta,7} - \mathrm{Tr}\,\gamma_{\theta,\bar{7}}\right) = -12\,. \tag{4.14}$$

As shown in appendix C, this orientifold singularity is not suitable to incorporate the standard model. Fortunately, $\mathbb{Z}_3$ fixed points other than the origin are not fixed under the orientifold action, and therefore are $\mathbb{C}^3/\mathbb{Z}_3$ *orbifold* singularities, which we may employ to embed local configurations as those in section 3.3.

The strategy is then to concentrate at an orbifold point different from the origin, say $(0,1,0)$ where we want to have the standard model (its $\mathbb{Z}_2$ mirror image $(0,-1,0)$ will have identical spectrum). We choose a suitable twist matrix for the D3-branes such as that in section 3.3. At these points the tadpole condition is simply:

$$\mathrm{Tr}\,\gamma_{\theta,7} - \mathrm{Tr}\,\gamma_{\theta,\bar{7}} + 3\,\mathrm{Tr}\,\gamma_{\theta,3} = 0\,. \tag{4.15}$$

The challenge to construct a model is to be able to accommodate the 32 D3-branes among the different orbifold/orientifold points such that the corresponding twist matrices satisfy the tadpole conditions (4.14) and (4.15) and of course having the same number of D7- and $\overline{\mathrm{D7}}$-branes. The other degree of freedom we have is the possibility of adding Wilson lines. They play an important role because at a given plane, a Wilson line differentiates among the different orbifold fixed points.

A Left-Right symmetric model. We can satisfy the tadpole condition (4.14) in many ways, but for future convenience we choose having eight D3-branes at the origin with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\alpha \mathrm{I}_4, \alpha^2 \mathrm{I}_4\right). \tag{4.16}$$

Let us now discuss the rest of the points $(0, n, p)$. Since these are only orbifold points the condition for tadpole cancellation is (4.15). If we want to have a standard-like model at a D3-brane on one of these points we must also have to introduce the D7- or $\overline{\mathrm{D7}}$-branes. Let us introduce $\mathrm{D7}_1$-branes at $Y_1 = 0$. In order not to alter the tadpole cancellation at the point $(0, 0, 0)$ the twist matrix for the 7-branes has to be traceless, so we take six $\mathrm{D7}_1$-branes with:

$$\gamma_{\theta,7} = \mathrm{diag}\left(\mathrm{I}_2, \alpha\ \mathrm{I}_2, \alpha^2\ \mathrm{I}_2\right). \tag{4.17}$$

In order to have non-trivial impact in the other points $(0, n, p)$ we add a Wilson line γ_W on the second complex plane. We choose

$$\gamma_{W,7} = \mathrm{diag}\left(\alpha, \alpha^2, \mathrm{I}_2, \mathrm{I}_2\right). \tag{4.18}$$

The addition of this Wilson line implies that $\mathrm{Tr}\,\gamma_{\theta,7}\gamma_W = \mathrm{Tr}\,\gamma_{\theta,7}\gamma_W^2 = -3$ so we need to have D3-branes only at the points $(0, n, p)$ with $n \neq 0$ with twist matrix satisfying $\mathrm{Tr}\,\gamma_{\theta,3} = 1$. We achieve this at each of the points $(0, \pm 1, 0)$ by adding seven D3-branes with twist matrix

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2\right). \tag{4.19}$$

The local structure at these points is like that of Left-Right models in section 3.3. Hence they will lead to a $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ gauge group with three quark-lepton generations.

Next consider the points $(0, \pm 1, \pm 1)$. We can easily cancel tadpoles by introducing one single D3-brane at each of the four points, with $\gamma_{\theta,3} = 1$

Finally we have to consider the points (m, n, p) with $m \neq 0$. So far we have six $\mathrm{D7}_1$-branes and $8 + 7 + 7 + 4 = 26$ D3-branes. Therefore we need to introduce six more D3-branes to complete 32, and six $\overline{\mathrm{D7}}$-branes. We can easily achieve this by locating three $\overline{\mathrm{D7}}_1$-branes at $Y_1 = \pm 1$ with twist matrix and Wilson lines given by

$$\gamma_{\theta,\bar{7}} = \mathrm{I}_3\,, \qquad \gamma_{\bar{W}\bar{7}} = \left(1, \alpha, \alpha^2\right). \tag{4.20}$$

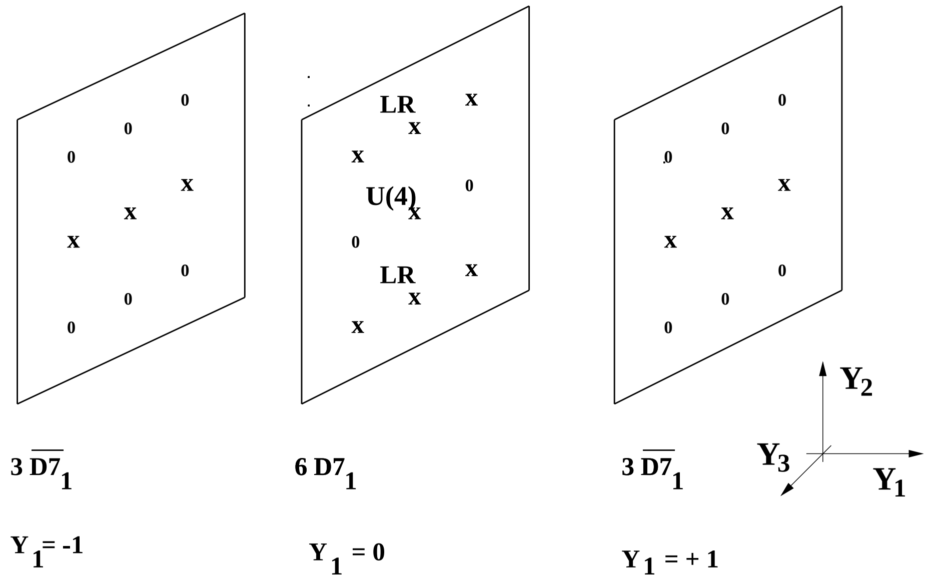

Figure 7: A compact $\mathbb{Z}_3$ orientifold model with no anti-D3-branes. A Left-Right symmetric sector is located at the fixed point $(0,1,0)$ and its orientifold mirror. There is a single D3-brane at each of the fixed points marked (x). At the origin, which is an orientifold point, there are eight D3-branes giving rise to a U(4) gauge group. The anti-D7-branes are trapped and cannot annihilate the D7-branes.

We then have $\mathrm{Tr}\,\gamma_{\theta,\bar{7}}\gamma_{\bar{W},\bar{7}} = \mathrm{Tr}\,\gamma_{\theta,\bar{7}}\gamma^2_{\bar{W},\bar{7}} = 0$ and do not need to add D3-branes at the points $(1,\pm1,p)$. At the points $(1,0,p)$ we can put a single D3-brane with $\gamma_{\theta,3} = 1$, canceling the tadpoles from $\gamma_{\theta,\bar{7}}$. At the points $(-1,n,p)$ we have a similar structure, consistently with the orientifold projection. In total, we have added precisely the six $\overline{\mathrm{D7}}$-branes and six D3-branes, as needed for consistency.

Since this model provides an interesting example of a realistic compactification with gravity mediated supersymmetry breaking, we give some details on the computation of the spectrum in the visible sector.

The D7-branes, with Chan-Paton embedding (4.17) would (before the orientifold projection) lead to a group $\mathrm{U}(2)^3$ with matter in three copies of $(2,2,1)+(1,2,2)+(2,1,2)$. The Wilson line (4.18) breaks the gauge symmetry to $\mathrm{U}(1)^2 \times \mathrm{U}(2) \times \mathrm{U}(2)$ with matter in three copies of $(2,\bar{2})$ with zero charge under the $\mathrm{U}(1)^2$. A simple way to read the spectrum is by using the shift vectors defined in the Cartan-Weyl basis (in analogy with [39]) for the twist matrix and Wilson line which in this case are:

$$V_{7_0} = \frac{1}{3}\,(0,0,1,1,-1,-1)\,, \qquad W_{7_0} = \frac{1}{3}\,(1,1,0,0,0,0)\,. \tag{4.21}$$

We keep states of the form $P = (1,-1,0,\ldots)$ (and permutations) satisfying $PV = 1/3$ and $PW = 0$. The orientifold projection identifies two pairs of groups and further breaks the symmetry to $\mathrm{U}(1)\times\mathrm{U}(2)$ with the simple matter consisting of three singlets (antisymmetrics of SU(2)) with charge $+2$ under the U(1) inside U(2).

The D3-branes at the origin, with Chan-Paton embedding (4.16) lead, before the orientifold projection, to an U(4) × U(4) gauge group with matter on three copies of $(4, \bar{4})$. After the orientifold projection this becomes a single U(4) group with matter on three copies of a **6**. For states in the **37** sector for D3-branes at the origin, we use the extended shift vector

$$\tilde{V} = \frac{1}{3}(1,1,1,1,-1,-1,-1,-1) \otimes V_{7_0} \tag{4.22}$$

with the first eight entries corresponding to the original $\mathrm{U}(4)^2$ on the D3-branes. We then keep the vectors[12] $P = (-1,0,\ldots;1,0,\ldots)$ satisfying $P\tilde{V} = 1/3$. We obtain the matter fields transforming as $(4,2)_0 + (\bar{4},1)_1 + (\bar{4},1)_{-1}$ under the final $\mathrm{U}(4) \times \mathrm{U}(2) \times \mathrm{U}(1)$.

For D3-branes at $(0,\pm 1,0)$ the Chan-Paton twist is (4.19), we obtain the spectrum of the LR model. The gauge group is $\mathrm{U}(3) \times \mathrm{U}(2) \times \mathrm{U}(2)$, with two anomalous U(1)'s being actually massive, and the diagonal combination giving $B{-}L$. In the 33 sector, we obtain matter fields

33 sector:

$$3\Big[(3,\bar{2},1) + (1,2,\bar{2}) + (\bar{3},1,2)\Big]\,. \tag{4.23}$$

These can be explicitly seen by using the effective shift vector $V_3 = \frac{1}{3}(0,0,0,1,1,-1,-1)$.

For matter in **37**, **73** sectors, the gauge quantum numbers are easily computed by considering the shift vectors $V_3 \otimes (V_7 \pm W_7)$, and keeping the state vectors $P = (-1,0,\ldots 0) \otimes (1,0\ldots,0)$ and permutations for the **37** sector, and the opposite sign for the **73** sector. We get:

37 sector:

$$(3,1,1;1)_{-1} + (3,1,1;2)_0 + (1,2,1;1)_1 + (1,2,1;2)_0 \tag{4.24}$$

73 sector:

$$(\bar{3},1,1;1)_1 + (\bar{3},1,1;2)_0 + (1,1,2;1)_1 + (1,1,2;2)_0\,. \tag{4.25}$$

The orientifold projection map the sets of branes at $(0,1,0)$ and $(0,-1,0)$ to each other, so only one copy of the LR model is obtained. Notice that these sector contain some extra vector-like chiral fields beyond those obtained in section 3.4. Six pairs of colour triplets will in general become massive once (7_i7_i) states get vevs. The remaining extra vector-like fields beyond the leptons transform like $(3,1,1)+(1,2,1)+(1,1,2)+$h.c.. As discussed in chapter 6, the presence of extra $\mathrm{SU}(2)_R$ doublets is in fact welcome in order to give rise of the required $\mathrm{SU}(2)_R$ gauge symmetry breaking.

[12]The **73** sector would have the opposite relative signs between the D7- and D3-branes; but it is identified to the **37** sector by the orientifold projection and need not be considered independently.

For D3-branes at fixed points $(0, \pm 1, \pm 1)$, with $\gamma_{\theta,3} = 1$, we have one U(1) gauge group, with no 33 matter, at each brane. The orientifold projection relates two pairs of these points reducing the group to $\mathrm{U}(1)^2$. The matter on each of these two 37 sectors can be easily computed as above (shift vector $0 \otimes V_7$) and transforms as $1_{-1,1} + 2_{0,1}$ under the $(\mathrm{U}(2) \times \mathrm{U}(1)) \times \mathrm{U}(1)$ (where the first U(1) arises from D7-branes and the second one from D3-branes). The 73 sector gives multiplets $1_{-1,-1} + 2_{0,-1}$.

A standard-like model. Let us now briefly present a model with a SM subsector. We put twelve D7-branes at $Y_1 = 0$ and four D3-branes at the origin with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2\right), \qquad \gamma_{\theta,7} = \mathrm{diag}\left(\alpha \mathrm{I}_6, \alpha^2 \mathrm{I}_6\right) \tag{4.26}$$

satisfying the tadpole condition (4.14) at the origin. We also introduce a Wilson line along the second plane, with action

$$\gamma_{W,7} = \mathrm{diag}\left(1, \alpha, \alpha^2, \mathrm{I}_9\right). \tag{4.27}$$

The contribution to twisted tadpoles for the orbifold fixed points $(0, \pm 1, 0)$ is given by $\mathrm{tr}\, \gamma_7 \gamma_W = \mathrm{tr}\, \gamma_7 \gamma_W^2 = 6\alpha^2 + 3\alpha$. It allows us to put six D3-branes at each of the two points with

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_1\right). \tag{4.28}$$

The structure at these points is of the SM type considered in section 3.3 (it differs from it by an irrelevant overall phase in the Chan-Paton embeddings). Hence we obtain one three-generation SM sector.

At each of the points $(0, \pm 1, \pm 1)$, we cancel the tadpoles by placing three D3-branes with $\gamma_3 = (\mathrm{I}_2, \alpha I_1)$ Finally, we put two D3-branes with $\gamma_3 = \mathrm{I}_2$ at each of the points $(0, 0, \pm 1)$. The total number of D3-branes we have introduced is $4 + 6 + 6 + 12 + 4 = 32$, and there are also twelve D7-branes.

The twelve $\overline{\mathrm{D7}}$-branes required to cancel the untwisted tadpole introduced by the D7-branes, could be evenly distributed at the positions $Y_1 = \pm 1$, with $\gamma_{\theta,\bar{7}} = (\mathrm{I}_2, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2)$. Being traceless, this embedding does not require to introduce more D3-branes nor Wilson lines. Unfortunately, these $\overline{\mathrm{D7}}$-branes can move off the fixed points into the bulk, and annihilate partially against the D7-branes at the origin. The result of such process is not necessarily disastrous for the SM sector we had constructed. In particular, it leads to a local structure of the type studied in section 3.6.5, leading to a non-supersymmetric SM with three generations. Hence the model would provide a realistic non-supersymmetric model, but necessarily requiring a TeV string scale.

Alternatively, in order to avoid the partial annihilation of branes and anti-branes and obtain a gravity mediated supersymmetry breaking scenario we can modify the model in a simple way. Let us add, for instance, a second Wilson line, now in the third plane, acting on the D7-branes with

$$\gamma_{W',7} = \left(\mathrm{I}_3; 1, \alpha, \alpha^2, 1, \alpha, \alpha^2, 1, \alpha, \alpha^2\right). \tag{4.29}$$

This Wilson line has no effect on the points $(0,\pm 1,0)$. Since $\mathrm{Tr}\,\gamma_{\theta,7}\gamma_{W',7}^{n}=3\alpha$ for $n=1,2$, we need to add two D3-branes at the points $(0,0,\pm 1)$ with $\gamma_{\theta,3}=\mathrm{diag}(1,\alpha^2)$ to cancel tadpoles. However, since $\mathrm{Tr}\,\gamma_{\theta,7}\gamma_{W,7}^{m}\gamma_{W',7}^{n}=0$ for $m,n=1,2$, we do not need to add any D3-brane at the points $(0,\pm 1,\pm 1)$. Therefore the remaining 12 D3-branes must sit at some of the fixed points $(\pm 1,m,n)$. We can achieve that having at each of the two planes $Y_1=\pm 1$ six $\overline{\mathrm{D7}}_1$-branes with $\gamma_{\theta,\overline{7}}=(\alpha\mathrm{I}_3,\alpha^2\mathrm{I}_3)$ with one Wilson line in the second plane with $\gamma_{W,\overline{7}}=(1,\alpha,\alpha^2,1,\alpha,\alpha^2)$ which precisely requires two D3-branes at each of the three fixed points points $(1,0,m)$ with $\gamma_3=(\alpha,\alpha^2)$ to cancel tadpoles. Putting a similar distribution at the plane $Y_1=-1$, we end up with a model in which the anti-branes are also trapped, they cannot leave the $Y_1=\pm 1$ planes. The modulus parameterizing this motion has been removed by the Wilson line We have then succeeded in constructing a standard-like model with antibranes trapped at a hidden sector located at $Y_1=\pm 1$ without any danger to annihilate with the D7-branes of the visible sector, so it is a genuine gravity-mediated scenario.

4.3 Inside a Calabi-Yau

Since our local singularities reproducing the semirealistic spectra in section 3 contain D7-branes, the general framework to discuss compact models is F-theory [45]. F-theory describes compactification of type-IIB string theory on curved manifolds in the presence of seven-branes. Cancellation of magnetic charge under the RR axion implies the compact models should in general include a set of (p,q) seven-branes, not mutually local, and around which the type-IIB complex coupling constant τ suffers non-trivial $\mathrm{SL}(2,\mathbb{Z})$ monodromies. Compactification to four dimensions on a three-complex dimensional manifold B_3, with a set of (p,q) seven-branes wrapped on two-complex dimensional hypersurfaces in B_3, can be encoded in the geometry of a four-fold X_4, elliptically fibered over B_3, with (p,q) seven-branes represented as the two-complex dimensional loci in B_3 over which the elliptic fiber degenerates by shrinking a (p,q) one-cycle. In order to preserve $\mathcal{N}=1$ supersymmetry in four dimensions, X_4 must be Calabi-Yau, while B_3 in general is not.

An important feature of four-dimensional compactifications of F-theory is that they generate a non-zero tadpole for the type-IIB 4-form A_4^+ [46], its value (in the absence of 3-form fluxes [47]) being given, in units of D3-brane charge, by $-\frac{1}{24}\chi(X_4)$, where $\chi(X_4)$ is the Euler characteristic of X_4. This tadpole may be cancelled by the addition of D3-branes (or instantons on the 7-brane gauge bundles, see below).

Hence, F-theory compactifications contain the basic ingredients employed in our local structures (namely D7-branes, D3-branes, and a non-trivial geometry B_3) in a completely supersymmetric fashion. Clearly, local structures of the type studied in section 3 will appear embedded in F-theory compactification where the base B_3 of the fourfold contains $\mathbb{C}^3/\mathbb{Z}_3$ singularities. In the following we construct a particular

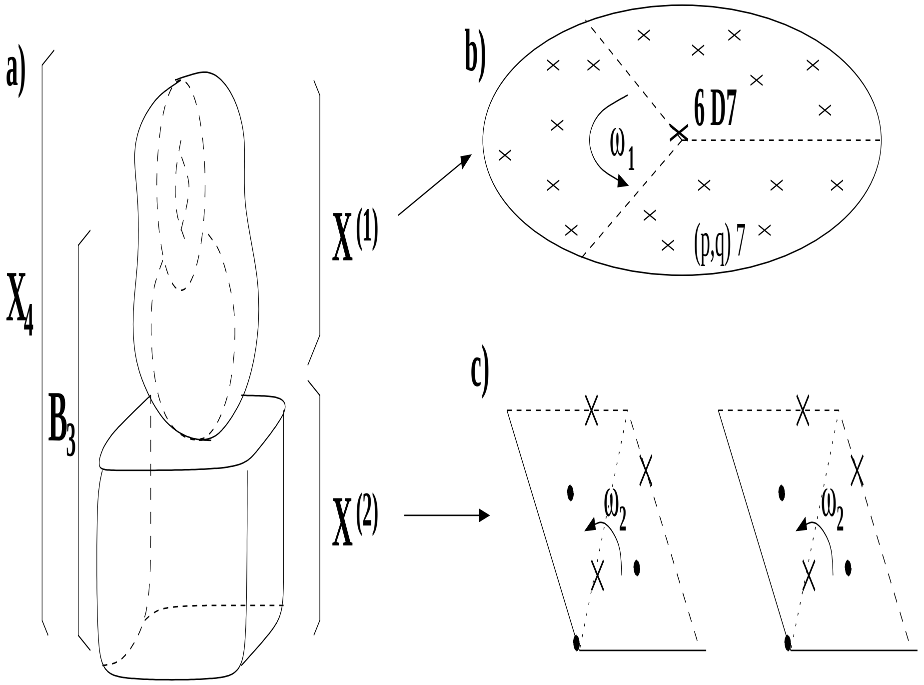

Figure 8: The fourfold X_4 is elliptically fibered over B_3, which is the product of $\mathbb{P}_1$ times the K3 $X^{(2)}$. Figure b) depicts the structure of the elliptic fibration over $\mathbb{P}_1$, with crosses denoting points at which the torus fiber degenerates. In type-IIB language, they correspond to seven-branes at points in $\mathbb{P}_1$ and wrapped over $X^{(2)}$. Six **D**7-branes sit at a $\mathbb{Z}_3$ invariant point, while the remaining 18 (p,q) seven-branes form a $\mathbb{Z}_3$ symmetric arrangement. Figure c) depicts the structure of $X^{(2)}$, which is a $\mathbb{T}^4/\mathbb{Z}_3$ orbifold, with fixed points denoted by dots. The final fourfold is obtained by quotienting X_4 by the combined $\mathbb{Z}_3$ action ω_1 and ω_2. Fixed points of w_2 in $X^{(2)}$ are denoted as crosses in figure c). The model yields a three-generation LR model arising from D3-branes sitting at the origin in $\mathbb{P}_1$ and at one of the ω_2 fixed points in $X^{(2)}$.

simple model containing a sector of D7-branes and D3-branes located at a $\mathbb{C}^3/\mathbb{Z}_3$ singularity, and reproducing the spectrum of a LR model of the type studied in section 3.4.

In order to keep the model tractable, we will take a simple fourfold as our starting point, namely $X_4 = X^{(1)} \times X^{(2)}$, where $X^{(1)}$ is an elliptically fibered K3, and $X^{(2)}$ is also a K3, which we will take to be a $\mathbb{T}^4/\mathbb{Z}_3$ orbifold. Unfortunately K3$\times$K3 cannot contain $\mathbb{C}^3/\mathbb{Z}_3$ singularities, hence our final model will be a $\mathbb{Z}_3$ quotient of X_4. The following construction of the model is pictured in figure 8

Let us start by describing X_4. The K3 manifold $X^{(1)}$ is a fibration of $\mathbb{T}^2$ over a $\mathbb{P}_1$, which can be defined in the Weierstrass form

$$y^2 = x^3 + f_8(z,w)x + g_{12}(z,w)\,, \tag{4.30}$$

where x, y parameterize $\mathbb{T}^2$, and $[z,w]$ are projective coordinates in $\mathbb{P}_1$. The functions f_8, g_{12} are of degree 8 and 12, respectively in their arguments. The elliptic fibration degenerates at the 24 points $[z, w]$ given by zeroes of the discriminant

$$\delta_{24} = 4f_8(z,w)^3 + 27g_{12}(z,w)^2 \,. \tag{4.31}$$

These degenerations signal the presence of 24 (p,q) seven-branes sitting at a point in $\mathbb{P}_1$ and wrapping $X^{(2)}$ completely. We will be interested in a particular family of K3 manifolds, containing six coincident **D**7-branes sitting at $z = 0$. Geometrically, we require the fibration (4.30) to have an I_6 Kodaira type degeneration [48] at $z = 0$, meaning f_8 and g_{12} are non zero at $z = 0$, but δ_{24} vanishes as z^6. We will be more explicit below.

For $X_4 = K3 \times K3$, $\chi(X_4) = 24 \times 24$, and the compactification leads to an A_4^+-tadpole of -24 units. It will be useful to rederive this result from a different point of view. On the world-volume M_7 of each seven-brane (of any (p,q) type) there exist couplings

$$-\frac{1}{24}\int_{M_7} R\wedge R\wedge A_4^+ + \int_{M_7} F\wedge F\wedge A_4^+ \,, \tag{4.32}$$

where R is the Ricci tensor of the induced curvature, and F is the field strength tensor for the world-volume gauge fields. In the absence of world-volume gauge instantons the second term drops, and the contribution of the first for each seven-brane wrapped on the K3 $X^{(2)}$ (recall $\chi(K3) = \int_{K3} R\wedge R = 24$) is $-\int_{M_4} A_4^+$, where M_4 is four-dimensional spacetime. Namely, we get an A_4^+-tadpole of -1 for each of the 24 seven-branes in the model, hence a total tadpole of -24. This tadpole can be cancelled by adding 24 D3-branes sitting at a point in $B_3 = \mathbb{P}_1 \times X^{(2)}$, or by turning on instantons on the seven-brane gauge bundles.

We will be interested in considering $X^{(2)}$ in the $\mathbb{T}^4/\mathbb{Z}_3$ orbifold limit. We start with $\mathbb{T}^4$ parametrized by two coordinates z_1, z_2 with identifications $z_i \simeq z_i + 1$, $z_i \simeq z_i + e^{2\pi i/3}$ (we set the radii to unity for the sake of simplicity), and mod out by the $\mathbb{Z}_3$ action

$$\theta : z_1 \longrightarrow e^{2\pi i/3} z_1 \,, \qquad z_2 \longrightarrow e^{-2\pi i/3} z_2 \,. \tag{4.33}$$

The quotient is a K3 manifold, flat everywhere except at the nine fixed points of θ (located at points (z_1, z_2) with z_1, $z_2 = 0$, $\frac{1}{3}(1 + e^{\pi i/3})$ or $\frac{1}{3}(e^{\pi i/3} + e^{2\pi i/3})$), which descend to singularities $\mathbb{C}^2/\mathbb{Z}_3$ in the quotient. The curvature is concentrated at those points, and gives a contribution of $\int_{C^2/\mathbb{Z}_3} R\wedge R = 8/3$ at each. Hence the -1 A_4^+-tadpole at each seven-brane is split in nine $-1/9$ A_4^+-tadpole, arising from the nine points $\mathbb{C}^2/\mathbb{Z}_3$ wrapped by the seven-brane world-volume. There are no contributions from world-volume gauge instantons if the gauge bundle over these $\mathbb{C}^2/\mathbb{Z}_3$ is trivial; in more familiar terms, if the $\mathbb{Z}_3$ action is embedded trivially on the seven-brane indices. Given this θ-embedding, one may worry about twisted tadpoles. However, since the $\mathbb{Z}_3$ twist leaves fixed the complete $\mathbb{P}_1$, such tadpoles receive contributions from all seven-branes, and they cancel by the same reason the overall axion RR-charge cancels, namely by cancellation among contributions of different (p,q) seven-branes.

As mentioned above, in order to obtain $\mathbb{C}^3/\mathbb{Z}_3$ singularities we are forced to consider a quotient of the four-fold X_4 considered above, by $\mathbb{Z}_3$, with generator ω acting simultaneously on $\mathbb{P}_1$ and $X^{(2)}$. We choose the action of ω on $\mathbb{P}_1$ to be given by

$$\omega_1 : z \longrightarrow e^{2\pi i/3} z\,, \qquad w \longrightarrow w\,. \tag{4.34}$$

For the configuration to be invariant under this action, the K3 $X^{(1)}$ must be given by (4.30) with the functions f_8, g_{12} depending only on z^3

$$\begin{aligned} f_8(z,w) &= Aw^8 + Bz^3w^5 + Cz^6w^2\,, \\ g_{12}(z,w) &= Dw^{12} + Ez^3w^9 + Fz^6w^6 + Gz^9w^3 + Hz^{12}\,. \end{aligned} \tag{4.35}$$

The requirement that there are six **D**7-branes at $z = 0$ amounts to the vanishing of the coefficients of order 1 and z^3 in δ_{24}

$$4A^3 + 27D^2 = 0\,, \qquad 12A^2B + 54DE = 0\,. \tag{4.36}$$

It will be convenient to require no seven-branes to be present at $[z,w] = [1,0]$, hence we require $H \neq 0$. The configuration we have constructed therefore contains six **D**7-branes at $z = 0$, and the remaining 18 seven-branes distributed on $\mathbb{P}_1$ in a $\mathbb{Z}_3$ symmetric fashion. Notice that not only the locations, but the (p,q) types of these seven-branes are consistent with the symmetry, since the F-theory K3 $X^{(1)}$ is $\mathbb{Z}_3$ invariant. Also, notice that no (p,q) seven-branes (other than the six at $z = 0$) are fixed under ω.

We have to specify also the action of ω on X^2, which we take to be

$$\omega_2 : z_1 \longrightarrow e^{2\pi i/3} z_1 + \frac{1}{3}(1 + e^{\pi i/3})\,, \qquad z_2 \longrightarrow e^{2\pi i/3} z_2 + \frac{1}{3}(1 + e^{\pi i/3})\,. \tag{4.37}$$

(The shifts have been included to avoid ω-fixed points to coincide with θ-fixed points.) That this is a symmetry of $X^{(2)} = \mathbb{T}^4/\mathbb{Z}_3$ follows from the fact that ω_2 is a symmetry of $\mathbb{T}^4$, and that it commutes with the action θ (4.33) required to form $\mathbb{T}^4/\mathbb{Z}_3$. The action ω_2 has nine fixed points in $\mathbb{T}^4$, given by points (z_1, z_2) with $z_i = \frac{1}{3}e^{\pi i/3}$, $\frac{1}{3}(1 + 2e^{\pi i/3})$, $\frac{1}{3}(e^{2\pi i/3} + 2e^{\pi i/3})$. The action of θ maps them to each other so that in the quotient they yield just three singular points. Similarly, ω_2 maps the nine fixed points of θ to each other, so that in the quotient they give rise to just three singularities. Notice that in the final model the $\mathbb{T}^4$ is modded out by θ and ω_2, and consequently by all other twists they generate. In this respect it is important to notice that $\theta\omega_2$ and $\theta^2\omega_2$ (and their inverses) are freely acting due to the shifts in (4.37), and do not generate new singularities.

The final model is obtained by quotienting $\mathbb{P}_1 \times X^{(2)}$ by ω, the combined action of (4.34) and (4.37). One can see that it preserves $\mathcal{N} = 1$ supersymmetry in the quotient by noticing that the holomorphic 4-form in X_4, given by $(dx/y)(dz/w)dz_1dz_2$ is invariant, hence the quotient is still a four-fold. Notice that the fixed points of ω correspond to $[z,w]=[0,1],[1,0]$ in $\mathbb{P}_1$ times the three fixed points in $X^{(2)}$.

We must also specify the action of ω on the seven-branes. For the (p,q) seven-branes at $z\neq 0$, the action is fully specified by the geometrical action in ω_1. The six **D**7-branes at $z=0$, sit at a fixed point of ω, and hence may suffer a non-trivial action on their Chan-Paton indices, which we choose to be given by

$$\gamma_{\omega,7}=\mathrm{diag}\left(e^{2\pi i\frac{1}{3}}\mathrm{I}_3, e^{2\pi i\frac{2}{3}}\mathrm{I}_3\right). \tag{4.38}$$

This amounts to choosing a non-trivial U(6) bundle on **D**7-brane world-volume. This will lead to a non-zero contribution to world-volume instanton number, and hence influence the computation of the A_4^+-tadpole in the quotient. Moreover, the embedding (4.38) also leads to non-zero ω-twisted tadpoles which must be properly cancelled.

Let us turn to the computation of the A_4^+ tadpole. Since the ω action has fixed points, the Euler characteristic of the quotient four-fold is *not* simply $\chi(X_4)/3$. Direct computation seems rather involved, hence we prefer to compute the A_4^+ tadpole by using the seven-brane world-volume couplings (4.32). The 18 (p,q) seven-branes at $z\neq 0$ are not invariant under ω, hence their world-volume $R\wedge R$ contribution comes only from the nine θ-fixed points, yielding a total A_4^+ tadpole of -18. This is reduced to -6 by the ω action (equivalently, the 18 seven-branes in X_4 descend to just six in the quotient by ω). The six **D**7-branes at $z=0$ sit at a ω fixed point, hence their $R\wedge R$ coupling receives contributions from three $\mathbb{C}^2/\mathbb{Z}_3$ (from nine θ-fixed ω-identified points), and three $\mathbb{C}^2/\mathbb{Z}_3$ (from nine ω-fixed θ-identified points). The total contribution is $-\frac{1}{24}\times\frac{8}{3}\times 3\times 2\times 6=-4$. There is a further contribution, from the instanton number of the non-trivial bundle implied by (4.38). This can be computed (see [26, 49] for a discussion of instanton numbers for orbifold spaces) to be $+1$ per $\mathbb{C}^3/\mathbb{Z}_3$ generated by ω, yielding an overall contribution $+3$ to the A_4^+ tadpole. Hence, the total A_4^+ tadpole in the model is -7, and one must introduce 7 D3-branes (as counted in the quotient) to achieve a consistent model.

Recall that the choice of the twist (4.38) generates non-zero ω-twisted tadpoles at the nine fixed points of ω_2 (three in the quotient), at $z=0$. Happily, these are $\mathbb{C}^3/\mathbb{Z}_3$ fixed points of the type studied in section 3.2, for which the twisted tadpole condition reads

$$\mathrm{Tr}\,\gamma_{\omega,7}+3\,\mathrm{Tr}\,\gamma_{\omega,3}=0\,. \tag{4.39}$$

The tadpoles are easily cancelled by choosing $\gamma_{\omega,3}=(I_3, e^{2\pi i\frac{1}{3}}I_2, e^{2\pi i\frac{2}{3}}I_2)$ at three of them (one after the θ-identification) and $\gamma_{\theta,3}=I_1$ at six (two in the quotient by θ). This employs 3 D3-branes (as counted in the quotient) out of the 7 we had available. The remaining 4 can be placed at a generic, smooth, point in the quotient (so that they would be described by $4\times 3\times 3$ in the covering space of the θ and ω actions).

This concludes the construction of the model. We have succeeded in constructing a $\mathcal{N} = 1$ supersymmetric compact model with a local singularity of the form $\mathbb{C}^3/\mathbb{Z}_3$, on which D3-branes and D7-branes sit. This local structure is of the kind considered in section 3.4, and leads to a Left-Right symmetric model $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)_{B-L}$ with three generations of standard model particles. Notice that in this concrete example the **D**7-brane gauge group is relatively large, but it should not be difficult to modify the construction (for instance, by adding Wilson lines on $\mathbb{T}^4$) to reduce it. We prefer not to complicate the discussion for the moment.

We would like to conclude with a remark. F-theory model building (see e.g. [7]) has centered on embedding the standard model gauge group on the seven-branes, while D3-branes are basically a useless sector. In our framework, the situation is inverted, with the D3-branes playing the main role, even if seven-branes also contribute to the spectrum. One particular advantage of our alternative embedding, which we have stressed throughout the paper, is that the relevant sector can be determined by using just the local behaviour of the compactification manifold. In this respect it is clear that many other consistent supersymmetric embeddings of our local structures may be achieved by considering more general four-folds with $\mathbb{C}^3/\mathbb{Z}_3$ singularities on the base, on which D3-branes and D7-branes (with gauge bundles locally reproducing the required monodromy) are located.

5. Some phenomenological aspects

Obtaining $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ (or its Left-Right symmetric extension $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$) and three quark-lepton generations in a simple manner is already remarkable. However we would like also to know if the class of models discussed in the previous sections is sufficiently rich to accommodate other important phenomenological features like gauge coupling unification, a hierarchical structure of quark/lepton masses and proton stability. In this chapter we would like to address these issues. Let us make clear from the beginning that we do not intend here to give a full account of these questions but rather to evaluate the phenomenological potential of these models without compromising ourselves with a particular one.

We will limit ourselves to the case of models obtained from D-branes at $\mathcal{N} = 1$ supersymmetric singularities. As already remarked above, some properties like number of generations, gauge group and gauge coupling normalization will mostly be controlled by the local D-brane configurations introduced in section 3. On the other hand other aspects like some Yukawa couplings or the $\mathcal{N} = 1$ Kahler metrics will be in general dependent on the particular compact space chosen in order to embed the realistic D-brane set. Thus, for example, the Kahler metric of the matter fields will in general be different for an embedding inside a compact $\mathbb{Z}_3$ orbifold compared to an embedding inside an F-theory model. Thus, the complete effective action will certainly depend on the specific model.

We will try to discuss certain features which seem generic among the class of models considered in the previous chapters. In particular, that is the case of the question of gauge coupling unification. We will also study some aspects of the Yukawa couplings both in the SM and LR type of models. As a general conclusion we believe that the structure of the models seems to be sufficiently rich to be able to accommodate the main phenomenological patterns of the fermion mass spectrum. Finally, we comment on the structure of mass scales and in particular the value of the string scale M_s that should be considered in these constructions. Since this depends on the compactification scale, the choice of M_s is also model dependent.

(i) Gauge coupling unification. This question obviously depends on whether we are considering a SM like the one considered in section 3.3 or a Left-Right symmetric model as in section 3.4. As we will see, from the gauge coupling unification point of view the Left-Right models look more interesting. Let us consider first the case of the SM. As mentioned in section 3, the tree-level gauge coupling constants at the string scale M_s are in the ratios $g_3^2 : g_2^2 : g_1^2 = 1 : 1 : 11/3$. The one-loop running between the weak scale M_Z and the string scale M_s is governed by the equations

$$\begin{aligned} \sin^2\theta_W(M_Z) &= \frac{3}{14}\left(1+\frac{11}{6\pi}\alpha(M_Z)\left(b_2-\frac{3}{11}b_1\right)\log\left(\frac{M_s}{M_Z}\right)\right), \\ \frac{1}{\alpha_3(M_Z)} &= \frac{3}{14}\left(\frac{1}{\alpha(M_Z)}-\frac{1}{2\pi}\left(b_1+b_2-\frac{14}{3}b_3\right)\log\left(\frac{M_s}{M_Z}\right)\right), \end{aligned} \tag{5.1}$$

where b_i are the β-function one-loop coefficients. Notice that the tree level result for $\sin^2\theta_W = 3/14 = 0.215$ is only slightly below the measured value at M_Z, $\sin^2\theta_W(M_Z) = 0.231$. Thus in order to get the correct sign for the one-loop correction we need to have $b_2 > 3b_1/11$. With the massless spectrum in table 1 we have $b_3 = 0$, $b_2 = 3$, $b_1 = 15$, although as mentioned in section 3.3, generically some three pairs of colour triplets will be heavy, leading to $b_3 = -3$, $b_2 = 3$, $b_1 = 13$. In both cases the one-loop correction has the wrong sign and one gets $\sin^2\theta_W(M_Z) = 0.18$, $M_s = 2.2\times10^{15}$ GeV ($\sin^2\theta_W(M_Z) = 0.204$, $M_s = 10^{10}$ GeV), respectively. Thus standard logarithmic gauge coupling unification within the particular SM configurations of section 3.3 does not seem to work. We cannot exclude, however that a more sophisticated configuration of D-branes (yielding, in particular, some extra massless doublets) could work [16, 50].

Let us now move to the Left-Right model case. In this case the gauge coupling unification question works remarkably well, as already pointed out in ref. [16]. Indeed, the massless spectrum of the $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}$ model of section 3.4 is essentially identical to the model considered in that reference. There are now two regions for the running, $M_R < Q < M_s$, where the gauge group is the Left-Right symmetric one and $M_Z < Q < M_R$ where the gauge group is the SM one. Thus M_R is the scale of breaking of the Left-Right symmetry. If the β-function coefficients of the Left-Right gauge group are denoted by B_3, B_L, B_R and B_{B-L}, the one-loop running yields

$$\sin^2\theta_W(M_Z) = \frac{3}{14}\left(1+\frac{11\alpha_e(M_Z)}{6\pi}\left[\left(B_L-\frac{3}{11}B_1'\right)\log\left(\frac{M_s}{M_R}\right)+\right.\right.$$
$$\left.\left.+\left(b_2-\frac{3}{11}b_1\right)\ \log\left(\frac{M_R}{M_Z}\right)\right]\right)\,, \quad (5.2)$$

$$\frac{1}{\alpha_e(M_Z)}-\frac{14}{3\alpha_3(M_Z)} = \frac{1}{2\pi}\left[\left(b_1+b_2-\frac{14}{3}b_3\right)\log\left(\frac{M_R}{M_Z}\right)+\right.$$
$$\left.+\left(B_1'+B_L-\frac{14}{3}B_3\right)\log\left(\frac{M_s}{M_R}\right)\right]\,, \quad (5.3)$$

where one defines

$$B_1' = B_R + \frac{1}{4}B_{B-L} \quad (5.4)$$

and b_i are the β-function coefficients with respect to the SM group. For the generic (i.e. no extra triplets) massless spectrum found in section 3.4 one has $B_3 = -3$, $B_L = B_R = +3$ and $B_{B-L} = 16$. To get a numerical idea let us assume that the Left-Right symmetry is broken not far away from the weak scale,[13] e.g. at $M_R \propto$ 1 TeV, and that the spectrum below that scale is given by that of the MSSM (so that $b_3 = -3$, $b_2 = 1$ and $b_1 = 11$).[14] Then one finds the results:

$$\sin^2\theta_W(M_Z) = 0.231\,; \qquad M_s = 9\times 10^{11}\,\mathrm{GeV} \quad (5.5)$$

in remarkable agreement with low-energy data. This agreement requires $\mathrm{SU}(2)_R$ to be broken not much above 1 TeV. In this connection, notice that the massless spectrum of the Left-Right model in section 3.4 does not have the required fields in order to do this breaking. However, slight variations like the explicit model in section 4.2.1 do have extra $\mathrm{SU}(2)_R$ doublets which can produce the breaking. On the other hand it is easy to check that the presence of the additional vector-like chiral fields in that model does not affect the one-loop coupling unification and hence the agreement remains.

In summary, although logarithmic gauge coupling unification in the SM would require some modification of the models, couplings unify nicely (with equal precision than in the MSSM) in the case of the $\mathrm{SU}(3)\times SU(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}$ models. In this case the string scale coincides with the unification scale and should be of order 10^{12} GeV.

[13] This is in fact the most natural assumption if both electroweak and $\mathrm{SU}(2)_R$ symmetry breaking are triggered by soft SUSY-breaking soft terms.

[14] A more general quantitative analysis for different values of M_R and SUSY-breaking soft terms may be found in ref. [16]. The general agreement is in general found as long as $M_R < 1$ TeV or so.

(ii) Yukawa couplings. Many phenomenological issues depend on the Yukawa coupling structure of the models. At the renormalizable level there are essentially three type of superpotential couplings involving physical chiral fields : a) $(33)^3$, b) (33)(73)(37) and c) (73)(37)(77). Whereas the couplings of types b) and c) depend on the structure of D7-branes and hence are more sensitive to global effects, that is not the case of couplings of type a) which involve couplings among different (33) chiral matter fields. Those are expected to depend mostly in the structure of the orbifold singularity on top of which the D3-branes reside. Indeed, for the more general case of a $\mathbb{Z}_3 \times \mathbb{Z}_M \times \mathbb{Z}_M$ singularity in the presence of discrete torsion (as in appendix A) the structure of those superpotentials has the form:

$$W = \sum_{i=0}^{2} \epsilon^{abc} \left[\Phi^a_{i,i+1} \Phi^b_{i+1,i+2} \Phi^c_{i+2,i}\right] + \left(1 - e^{2\pi i \frac{1}{M}}\right) s^{abc} \left[\Phi^a_{i,i+1} \Phi^b_{i+1,i+2} \Phi^c_{i+2,i}\right] , \quad (5.6)$$

where (see eq. (A.5)) s^{abc} is such that $s^{321} = s^{132} = s^{213} = 1$ and all other components vanish. The gauge group is $\mathrm{U}(n_0) \times \mathrm{U}(n_1) \times \mathrm{U}(n_2)$ and the $\Phi^a_{i,j}$ are bifundamental representations $(n_i, \overline{n_j})$. In the absence of discrete torsion the second piece vanishes and the superpotentials are purely antisymmetric. In the particular case of the SM, these include superpotential couplings corresponding to Yukawa couplings giving masses to U-type quarks, i.e. couplings of type $h_{abc} Q^a_L U^b_R H^c$. The corresponding Yukawa couplings have thus the form:

$$h_{abc} = \epsilon_{abc} + \left(1 - e^{2\pi i \frac{1}{M}}\right) s_{abc} \quad (5.7)$$

with $a, b, c = 1, 2, 3$ are family indices. However, to make contact with the low energy Yukawa couplings we have to recall that in a general $\mathcal{N} = 1$ supergravity theory the kinetic terms of these chiral fields will not be canonical but will be given by a a Kahler metric K_{ab} which will in general be a function of the compactification moduli. Consider, to be specific, the case in which the compact case is a standard $\mathbb{Z}_3$ toroidal orbifold or orientifold, like the examples shown in chapter 4. In this case one has $K_{ab} = (T_{a\bar{b}} + T^*_{a\bar{b}})^{-1}$, where $T_{a\bar{b}}$ are the nine untwisted Kahler moduli of the $\mathbb{Z}_3$ orbifold. Now, one can easily check that the low-energy physical Yukawa couplings corresponding to canonically normalized standard model fields h^0_{abc} will be related with the supergravity base ones h_{abc} as [51]:

$$h^0_{abc} = h_{lmn} \exp\left(\frac{K}{2}\right) (K_{al} K_{bm} K_{cn})^{-1/2} , \quad (5.8)$$

where we have neglected a complex phase irrelevant for our purposes and K denotes the full Kahler potential. The latter is given in the weak coupling limit by the expression $K = -\log(S + S^*) - \log\ \det(T_{a\bar{b}} + T^*_{a\bar{b}})$, where S is the complex dilaton field. Thus altogether, the low energy physical Yukawa couplings of U-type quarks in these models will have the general form:

$$h^0_{abc} = (S + S^*)^{-1/2} \left[\epsilon_{abc} + \left(1 - e^{2\pi i \frac{1}{M}}\right) s^{lmn} \det(t_{jk})^{-1/2} (t_{al} t_{bm} t_{cn})^{1/2}\right] , \quad (5.9)$$

where we define $t_{ab} = T_{a\bar{b}} + T^*_{a\bar{b}}$. Notice that the Yukawa couplings are proportional to the gauge coupling constant g, since $ReS = 2/\lambda$, λ being the type-IIB dilaton. Also, in the absence of discrete torsion ($s^{abc} = 0$) these Yukawa couplings do not depend on the moduli.

To see whether this kind of structure has a chance to describe the observed pattern of U-quark masses, let us consider the case in which only one of the three Higgs fields, say H_1, eventually gets a vev of order the weak scale, $\langle H_1 \rangle = v$. The U-quark mass matrix will be thus given by $M^U_{bc} = vh^0_{1bc}$. Let us assume also that the moduli with largest vevs are t_{11}, t_{33} and the off-diagonal ones t_{23}, t_{32}. Then the U-quark mass matrix would have the general form in leading order:

$$M^U_{bc} = (S + S^*)^{-1/2} v \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & h^0_{123} \\ 0 & h^0_{132} & h^0_{133} \end{pmatrix} . \tag{5.10}$$

Plugging the above expressions for the Yukawa couplings one finds for $|t_{33}| \gg |t_{32}|$ three U-quark eigenstates with masses of order $gv(0, 1/\delta, \delta)$, with $\delta = |t_{33}/t_{32}|^{1/2}$, yielding a hierarchical structure for the U-quarks. Thus the structure of the U-quarks Yukawa couplings for this class of models is in principle sufficiently rich to accommodate the observed hierarchy, at least as long as off-diagonal moduli have modest hierarchical vevs. In the particular setting of toroidal $\mathbb{Z}_3$ orbifolds discrete torsion seems also to be required.

The same discussion applies directly to the case of the Left-Right symmetric model of section 3.4, the only difference being that in this case the Yukawa couplings $h_{abc} Q^a_L Q^b_R H^c$ give masses both to U-quarks and D-quarks. For both a hierarchical structure should be possible to be accommodated.

The rest of the phenomenologically relevant renormalizable Yukawa couplings are of type b) and c), i.e. they involve directly the D7-brane sectors and are thus more compactification dependent. Let us describe some qualitative features for the SM and the LR models in turn.

In the case of the SM of section 3.3 (and its compact versions), there are renormalizable couplings of type $(33)_i(7_i3)(37_i)$, where $(33)_i$ denotes the chiral fields in the (33) sector associated to the i-th complex plane. Looking at the quantum numbers in table 1 one sees that in particular there are couplings of type $(3, 2)(\bar{3}, 1; 2')(1, 2; 2')$ which provide Yukawa couplings for the D-quarks. One of the doublets inside $(1, 2; 2')$ should be identified with the normal left-handed leptons and the other with Higgs fields. By definition, the doublet appearing in the D-quark Yukawa coupling will be identified with the Higgs field. As pointed out in the footnote at the end of section 3.2, in generic D7-brane configurations fields from a given 37 sector can couple to 33 fields from all three complex planes. Thus this flexibility will also in general allow for a hierarchy of D-quark masses. Concerning lepton masses, they do not appear at the renormalizable level since all left- and right-handed leptons as well as

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the Higgs field belong to (37_i) sectors and there are no $(37)^3$ type of couplings on the disk. They may however appear from non-renormalizable couplings of type $(37)^n$ in compact models if some standard model singlets in some other (37) sectors get a vev. This will be very model dependent.

In the case of the Left-Right symmetric model of section 3.4 (and its compact versions), the $(33)^3$ type of couplings already give masses to both U- and D-quarks. Concerning leptons, all anomaly free gauge symmetries allow for couplings of type (33)(37)(73) including terms transforming like $(1,2,2)(1,2,1)(1,1,2)$ which would give rise to standard Dirac masses for leptons. Looking at table 2 one observes that those couplings are in fact forbidden by some anomalous U(1) charges. Nevertheless one expects those couplings to be present once appropriate twisted closed string insertions (which are charged under anomalous U(1)'s) are made [16]. Again, for different geometrical configurations of D7-branes there will be enough flexibility to obtain a hierarchy of Yukawa couplings also for leptons.

In the case of the Left-Right model we also have to worry about neutrino masses. Generically they will get Dirac masses of the order of the charged lepton masses. However, once $SU(2)_R$ symmetry-breaking takes place through the vevs of the extra $SU(2)_R$ doublets discussed above, the right-handed neutrinos may combine with some singlets in (7_i7_j) sectors and get masses of order M_R. We will not give details here which would be very model dependent. Let us note however that the effective left-handed neutrino masses would thus be of order m_l^2/M_R, where m_l are the neutrino Dirac masses. For values of m_l of order the electron mass and M_R of order 1 TeV, they would have masses of order 1 eV, compatible with experimental bounds.

Let us finally comment about proton stability in these schemes. This is an important question if, as seems to be indicated by gauge coupling unification, the string scale is chosen to be well below the Planck mass (e.g. at scales of order 10^{12} GeV or below). Since for this question higher-dimensional operators are in principle important, it is not possible to make model independent statements about sufficient proton stability. We would like to underline however, that from this point of view the Left-Right symmetric models seem more promising since the symmetry $B-L$ is gauged and hence dimension four baryon or lepton violating operators are forbidden to start with. Furthermore, B/L-violating dimension 5 operators like F-terms of the form $Q_LQ_LQ_LL$ (which respect $B-L$) are of type $(33)^3(73)$ and hence are forbidden on the disk. In fact, it was shown in ref. [16], that in a Left-Right model analogous to the compact orientifold of chapter 5, the proton is absolutely stable due to the presence of induced baryon and lepton parity $\mathbb{Z}_2$ symmetries. Thus we believe that, at least in the Left-Right class of models, a sufficiently stable proton may naturally be achieved.

(iii) String scale versus planck mass. In our approach, four-dimensional gravity arises once we embed our SM or LR D-brane systems into a finite compact space. The precise relationship between the string scale M_s and four-dimensional Planck scale M_p will depend on the compactification. In toroidal orbifold compactifications with factorized two-tori they are related by

$$M_p = \frac{2\sqrt{2}M_s^4}{\lambda M_1 M_2 M_3}\,, \tag{5.11}$$

where M_i are the compactification mass scales of the three tori. Thus, as is well known [5, 52, 53, 54], values for M_s much below the Planck scale may be possible if some of the compact dimensions are very large. Until we get a handle on the dynamics fixing the radii, the value of the string scale is only constrained by accelerator limits to be $M_s > 1\,\mathrm{TeV}$ or so. In particular, one can accommodate intermediate scale values [55, 56] of order $M_s \propto 10^{10} - 10^{12}\,\mathrm{GeV}$, as suggested by the logarithmic gauge coupling unification arguments above.

Similar qualitative statements can be made concerning the Planck versus string scales in F-theory models with the SM sector living on the world-volume of D3-branes, like those discussed in chapter 5. Thus, e.g. one can match the observed size of the Planck scale by making very large the volume of the base P_1 in the first K3 $X^{(1)}$, while maintaining the volume of the second K3, $X^{(2)}$ of order the string scale.

It is quite interesting to remark that such intermediate values for the string scale turn out to be also suggested in a completely independent manner in models [14, 16] containing anti-branes like the orientifolds in chapter 5. Indeed, in those models supersymmetry is broken at the string scale M_s due to the presence of anti-D7-branes. However the physical SM or LR sectors are away from these antibranes in the Y_1 transverse dimension. Let us suppose that e.g. one has $M_2, M_3 = M_s$ and $M_1 = M_s^2/(\lambda M_p)$ so that one obtains the correct Planck scale. This means taking very large Y_1 radii which will also mean that the SUSY-breaking effects from the anti-D7-branes will be felt in a Planck mass suppressed manner in the visible SM world. Thus one expects effective SUSY-breaking contributions of order M_s^2/M_p in the SM sector. If M_s is of order $10^{10} - 10^{12}\,\mathrm{GeV}$, then these SUSY-breaking contributions would be of order the weak scale, as required. The structure of these models would be very similar to hidden-sector supersymmetry-breaking models, the main difference being that in the present case the string and SUSY-breaking scales do coincide.

Let us also comment that the approach here presented is also consistent with low (i.e. 1-10 TeV) string scale scenarios [53, 54] , just by choosing larger compact radii. In these cases an alternative understanding of the gauge coupling unification problem should be found.

Finally the F-theory constructions of the previous section allows for the possibility of a more standard scenario. Being supersymmetric, we do not have to identify the string scale with the supersymmetry breaking scale. We may in principle conceive models of this type for which the string scale is closer to the Planck or GUT scales.

The Left-Right symmetric model presented in that section still requires unification at the intermediate scale but in general, achieving or not gauge coupling unification will clearly depend on the spectrum of each model.

6. Final comments and outlook

We have presented what we consider to be the first steps on a new way to deal with string model building. The fact that D3-brane worlds can appear naturally on type-IIB string theory suggests that looking directly to the physics on these D3-branes may tell us many interesting phenomenological aspects that depend very weakly on the details of the compactified space. This is so since only gravitational strength interactions feel the extra dimensions. It would be interesting to extend this bottom-up construction of realistic models to other string theories, for instance using type-0B D3-branes on singularities [60], or to M-theory. The latter framework was explored in [61], where an explicit M5-brane configuration led to a $N = 2$ supersymmetric (and therefore non-chiral) toy model with SM group and correct gauge non abelian and U(1) quantum numbers for some of the standard model fields.

It is remarkable how easy it is to obtain realistic models from this approach and how powerful the structure of singularities turn out to be in order to achieve interesting phenomenological properties, such as the number of families, the ubiquity of hypercharge and the value of Weinberg's angle. For instance a concrete prediction of the class of models considered is that the number of families does not exceed 3 for supersymmetric models and 4 for non-supersymmetric ones. The reason being essentially the $\mathcal{N} = 4$ symmetry underlying the **33** spectrum, which limits the number of replicated fermions after the orbifold projection. This is a very small (and realistic) number compared with more standard compactifications in which the number of families depends on the topology of the compact manifold and is naturally very large.

It is worth remarking that the consideration of the non-compact models constructed here may have an importance *per se*. At present there is increasing interest on non-compact extra dimensions raised after the work of Randall and Sundrum [57]. It would be tempting to speculate that a similar mechanism to localize gravity on the D3-branes may be present in models similar to ours. However it is not clear if this could be achievable. Furthermore, during the past few years, non-compact brane models were used in order to obtain information on the dynamics of gauge theories from the structure of branes. Much work has been done in this direction, [58], although most of the activity was based on extended supersymmetric models and non-chiral $\mathcal{N} = 1$ models. Chiral theories were constructed in [59], and seen to be related to theories on D3-branes at singularities [21]. Our models extend this program by addressing (and actually succeeding in) the construction of realistic models using brane configurations.

Independent of these issues, the fact that in compact models we have been able to obtain realistic $\mathcal{N}=0$ and $\mathcal{N}=1$ models from type-IIB strings at singularities may be the most important concrete result of this work. We have explored several mechanisms and generalizations to current model building, each of them deserving further study. The new approach, while sharing some of the good properties of the constructions of [14, 16] (gauge unification, supersymmetry breaking, proton stability), has several clear advantages over previous discussions, besides the fact that it is more general: from the stability of the models, by trapping anti-branes avoiding brane/anti-brane annihilation, to the inclusion of new degrees of freedom, such as discrete torsion, which allow more flexibility on the structure of fermion masses and helps to stabilize the models.

A scenario with intermediate fundamental scale is clearly favoured by the non-supersymmetric models which can also have gauge coupling unification at the same scale, especially in the case of the Left-Right symmetric models. On the other hand, the fact that we have also obtained quasi-realistic $\mathcal{N}=1$ supersymmetric models allows for the possibility of realistic scenarios with fundamental scales as large as the Planck or GUT scales. It is worth pointing out however that, as usual in string theory, grand unified gauge models do not seem to appear naturally in this scheme, whereas the standard model and its Left-Right symmetric extension are very easy to obtain.

There are clearly many avenues that deserve to be explored in the future, starting from our approach. Probably the most pressing one being the understanding of moduli and dilaton stabilization after supersymmetry breaking and the corresponding value of the vacuum energy.

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A. Orbifold singularities with discrete torsion

Consider a $\mathbb{C}^3/(\mathbb{Z}_{M_1}\times\mathbb{Z}_{M_2})$ orbifold singularity, with the two factors of the orbifold group generated by twists g_1, g_2. The effect of discrete torsion in the closed string sector was analyzed in [62] (see also [63, 64]). It amounts to the introduction of an additional phase $\epsilon(g_1, g_2)$ in the action of g_1 on g_2-twisted sector states. This phase must be an s-th root of unity, where $s = \gcd(M_1, M_2)$. Hence discrete torsion modifies the twisted sector spectrum, and introduces relative phases among the different contributions to the torus partition function.

The effect of discrete torsion on open string sector, and consequently on systems of D-branes at singularities, has been analyzed only recently [33, 65] (see [66] for orientifold examples). In the presence of discrete torsion the embedding of the orbifold twists in the Chan-Paton indices forms a projective representation of the orbifold group [33], i.e. the Chan-Paton embedding matrices γ_g obey the group law up to phases, concretely

$$\gamma_{g_1}\gamma_{g_2} = \epsilon(g_1, g_2)\gamma_{g_2}\gamma_{g_1} \,. \tag{A.1}$$

The field theory on a set of D3-branes at such an orbifold singularity with discrete torsion has been discussed in detail in [33, 65]. In the following we center on a particular case which illustrates the main features.

Consider a $\mathbb{C}^3/(\mathbb{Z}_N \times \mathbb{Z}_M \times \mathbb{Z}_M)$ singularity, where the generator θ of $\mathbb{Z}_N$ acts on $\mathbb{C}^3$ through the twist $(a_1, a_2, a_3)/N$, and the generators ω_1, ω_2 of the $\mathbb{Z}_M$'s act through the twists $v_1 = (1, 0, -1)/M$, $v_2 = (0, 1, -1)/M$.[15] Let us consider the case of discrete torsion $\epsilon(\omega_1, \omega_2) = e^{-2\pi i/M}$ in the $\mathbb{Z}_M \times \mathbb{Z}_M$ part. Consequently we choose the following embedding in the Chan-Paton indices:

$$\begin{aligned}
\gamma_{\theta,3} &= \mathrm{diag}\left(I_{Mn_0}, e^{2\pi i\frac{1}{N}} I_{Mn_1}, \ldots, e^{2\pi i\frac{N-1}{N}} I_{Mn_{N-1}}\right) , \\
\gamma_{\omega_1,3} &= \mathrm{diag}\left(1, e^{2\pi i\frac{1}{M}}, \ldots, e^{2\pi i\frac{M-1}{M}}\right) \otimes \mathrm{diag}\left(I_{n_0}, I_{n_1}, \ldots, I_{n_{N-1}}\right) , \\
\gamma_{\omega_2,3} &= \begin{pmatrix} & 1 & & \\ & & \cdots & \\ & & & 1 \\ 1 & & & \end{pmatrix} \otimes \mathrm{diag}\left(I_{n_0}, I_{n_1}, \ldots, I_{n_{N-1}}\right) .
\end{aligned} \tag{A.2}$$

Notice that $\gamma_{\omega_1,3}\gamma_{\omega_2,3} = e^{-2\pi i\frac{1}{M}}\gamma_{\omega_2,3}\gamma_{\omega_1,3}$, in agreement with (A.1).

The spectrum is obtained after imposing the projections

$$\begin{array}{llll}
\text{Vector} & \lambda = \gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1} \,, & \lambda = \gamma_{\omega_1,3}\lambda\gamma_{\omega_1,3}^{-1} \,, & \lambda = \gamma_{\omega_2,3}\lambda\gamma_{\omega_2,3}^{-1} \\
\text{Chiral}_1 & \lambda = e^{2\pi i\frac{a_1}{N}}\gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1} \,, & \lambda = e^{2\pi i\frac{1}{M}}\gamma_{\omega_1,3}\lambda\gamma_{\omega_1,3}^{-1} \,, & \lambda = \gamma_{\omega_2,3}\lambda\gamma_{\omega_2,3}^{-1} \,, \\
\text{Chiral}_2 & \lambda = e^{2\pi i\frac{a_2}{N}}\gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1} \,, & \lambda = \gamma_{\omega_1,3}\lambda\gamma_{\omega_1,3}^{-1} \,, & \lambda = e^{2\pi i\frac{1}{M}}\gamma_{\omega_2,3}\lambda\gamma_{\omega_2,3}^{-1} \,, \\
\text{Chiral}_3 & \lambda = e^{2\pi i\frac{a_3}{N}}\gamma_{\theta,3}\lambda\gamma_{\theta,3}^{-1} \,, & \lambda = e^{-2\pi i\frac{1}{M}}\gamma_{\omega_1,3}\lambda\gamma_{\omega_1,3}^{-1} \,, & \lambda = e^{-2\pi i\frac{1}{M}}\gamma_{\omega_2,3}\lambda\gamma_{\omega_2,3}^{-1} \,.
\end{array} \tag{A.3}$$

The projection process is relatively simple. For vector multiplets, the θ projection yields a gauge group $\prod_{i=0}^{N-1} \mathrm{U}(Mn_i)$. The ω_1 projection splits it into $\prod_{i=0}^{N-1} \mathrm{U}(n_1)^N$. Finally, the ω_2 action identifies the different gauge factors with equal rank, leaving

[15] In certain cases, for instance when N and M are coprime, the group is actually $\mathbb{Z}_{NM} \times \mathbb{Z}_M$. However, it will be convenient to make the $\mathbb{Z}_M \times \mathbb{Z}_M$ action more manifest.

a remaining gauge group $\prod_{i=0}^{N-1} \mathrm{U}(n_i)$. Proceeding analogously, the complete spectrum is

$$\begin{aligned} &\text{Vector } \prod_{i=0}^{N-1} \mathrm{U}(n_i)\,, \\ &\text{Chiral } \sum_{r=1}^{3} \sum_{i=0}^{N-1} (n_i, \overline{n}_{i+a_r})\,. \end{aligned} \tag{A.4}$$

Notice this is identical to the spectrum of the $\mathbb{C}^3/\mathbb{Z}_N$ singularity with twist $(a_1, a_2, a_3)/N$, in section 2.1 or 3.1. However, the effect of the additional $\mathbb{Z}_M \times \mathbb{Z}_M$ twist and of the discrete torsion are manifest in the superpotential, which reads

$$\begin{aligned} W = \mathrm{Tr}\Big[&\Phi^1_{i,i+a_1} \Phi^2_{i+a_1,i+a_1+a_2} \Phi^3_{i+a_1+a_2,i} - \\ &- e^{-2\pi i \frac{1}{M}} \Phi^1_{i,i+a_1} \Phi^3_{i+a_1,i+a_1+a_3} \Phi^2_{i+a_1+a_3,i} \Big] \end{aligned} \tag{A.5}$$

which differs from the **33** piece in (3.1).

We spare the reader the detailed discussion of the introduction of D7-branes in the configuration. The resulting spectrum is identical to that obtained for $\mathbb{Z}_N$ singularities in section 3.1.

Notice that, since the twists ω_1, ω_2 have traceless Chan-Paton embeddings, the corresponding disk tadpoles vanish. The only constraints on the integers n_i arise from cancellation of tadpoles in $\mathbb{Z}_N$ twisted sectors. The corresponding conditions are those for $\mathbb{Z}_N$ singularities (as expected, since they are basically the anomaly cancellation conditions).

B. Non-abelian orbifolds

The rules to compute the spectrum of D3-branes at non-abelian orbifold singularities have been discussed in [67] for $\Gamma \subset \mathrm{SU}(2)$, and generalized in [20]. In the following we center on $\mathcal{N} = 1$ supersymmetric field theories in the case $\Gamma \subset \mathrm{SU}(3)$, studied in detail in [34, 35] (see [68] for the general case $\Gamma \subset \mathrm{SU}(4)$).

The computation of the spectrum is just a natural group-theoretical generalization of the abelian case. Let $\{\mathcal{R}_i\}_{i=1,\ldots,N}$ denote the irreducible representations of Γ, with N the number of conjugacy classes of Γ. We are interested in the field theory on a set of D3-branes at a $\mathbb{C}^3/\Gamma$ singularity, where the action of Γ on $\mathbb{C}^3$ is specified by a three-dimensional representation $\mathcal{R}^{(\mathbf{3})}$, and its action on the D3-brane Chan-Paton indices is given by a representation $\mathcal{R}^{CP}$, which decomposes as

$$\mathcal{R}^{CP} = \sum_{i=1}^{N} n_i \mathcal{R}_i\,. \tag{B.1}$$

The field theory on D3-branes at $\mathbb{C}^3/\Gamma$ is obtained by projecting the field theory on D3-branes on $\mathbb{C}^3$ onto states invariant under the combined (geometric plus Chan-Paton) action of Γ. The resulting gauge group[16] is $\prod_{i=1}^{N} \mathrm{U}(n_i)$. There are also $a_{ij}^{\mathbf{3}}$ chiral multiplets in the $(n_i, \overline{n}_j)$ representation, where the adjacency matrix $a_{ij}^{\mathbf{3}}$ is defined by the decomposition

$$\mathcal{R}^{(\mathbf{3})} \otimes \mathcal{R}_i = \sum_{j=1}^{N} a_{ij}^{\mathbf{3}} \mathcal{R}_j \,. \tag{B.2}$$

The superpotential is obtained by substituting the surviving chiral multiplets Φ_{ij} in the $\mathcal{N}=4$ superpotential (2.1), but we will not need it explicitly.

An important observation is that the gauge couplings for the different factors are not equal in field theories from non-abelian orbifolds. The gauge coupling for the i-th group is given by [20]

$$\tau_i = \frac{r_i \tau}{|\Gamma|} \tag{B.3}$$

with $r_i = \dim \mathcal{R}_i$, and τ an overall value independent of i.

The analysis of U(1) anomalies has not been performed in the literature, but can be easily generalized from the abelian case[17] Assuming the 37, 73 sectors contribute a set of bifundamental multiplets which cancel the non-abelian anomaly in the 33 sector, the mixed Q_{n_i}-$\mathrm{SU}(n_j)^2$ anomaly reduces to $A_{ij} = \frac{1}{2} n_i (a_{ij}^{(\mathbf{3})} - a_{ji}^{(\mathbf{3})})$. It is possible to check that the diagonal combination

$$Q_{\mathrm{diag}} = \sum_{i=1}^{N} \frac{r_i}{n_i} Q_{n_i} \tag{B.4}$$

is automatically non anomalous. Also, as long as no n_i vanishes, there is one additional non-anomalous U(1)'s for each non-trivial conjugacy classes leaving some complex plane invariant. An important difference with respect to the abelian case is that, due to the presence of the factors r_i above, the structure $\mathrm{SU}(3) \times \mathrm{SU}(2)$ does *not* in general guarantee the correct hypercharge assignments for the non-anomalous U(1)'s.

C. Non-orbifold singularities

In this section we briefly discuss systems of D3-branes at non-orbifold singularities. The only known systematic approach to construct the world-volume field theory is to realize the non-orbifold singularity as a partial resolution of a suitable orbifold

[16] In the literature on D-branes on non-abelian orbifold, the representation $\mathcal{R}^{CP}$ is usually taken to be k copies of the regular representation, $n_i = k \dim \mathcal{R}_i$, which satisfies tadpole cancellation conditions without the need of additional branes. We will consider more general choices of n_i, with the understanding that additional D7-branes will cancel the corresponding tadpoles. Unfortunately, the inclusion of D7-branes has not been discussed in the literature, hence we will not be explicit about the 37 sector in our models, and simply assume it provides additional chiral multiplets to cancel gauge theory anomalies from the 33 sector.

[17] We thank A. Hanany for conversations on this point.

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singularity [30].[18] However, the identification of the field-theory interpretation of the partial blowing-up is very involved beyond the simplest examples [30, 74, 75]. Consequently, it is difficult to make general statements about the phenomenological potential of these systems.

We would like to center on a particular family of singularities, which can be constructed as orbifolds of the conifold singularity [71] (see also [76]), and their partial blow-ups. They have potential interest since generically they lead to chiral $\mathcal{N}=1$ supersymmetric field theories on the world-volume of D3-branes, and include abelian orbifold singularities as particular cases. Moreover, they have a T-dual representation in terms of configurations of NS-fivebranes and D5-branes, known as "brane diamonds" [77], generalizing the brane box constructions in [59, 21], which allow a systematic search of interesting field theories within this class.

Instead of extending on such analysis, we point out that the only singularity in this family, leading to full triplication of chiral multiplet content in the 33 sector is in fact the $\mathbb{C}^3/\mathbb{Z}_3$ singularity. A milder requirement would be to have triplication not for the full 33 sector, but at least for some representation. Besides the $\mathbb{Z}_3$ orbifold, there is only one singularity in this family fulfilling this milder requirement. Let us briefly discuss its construction.

Consider the conifold, given by the hypersurface $xy = zw$ in $\mathbb{C}^4$. The field theory on D3-branes at a conifold was determined in [69]. The gauge group is $\mathrm{U}(N_1)\times\mathrm{U}(N_2)$ and there are chiral multiplets A_1, A_2 in the $(\square, \overline{\square})$, and B_1, B_2 in the $(\overline{\square}, \square)$. There is also a superpotential

$$W = \mathrm{Tr}(A_1B_1A_2B_2) - \mathrm{Tr}(A_1B_2A_2B_1)\,. \tag{C.1}$$

We are going to consider a quotient of this variety by a $\mathbb{Z}_3$ twist with generator θ acting as

$$x \longrightarrow e^{2\pi i/3}x\,; \qquad z \longrightarrow e^{2\pi i/3}z\,; \qquad y \longrightarrow e^{-2\pi i/3}y\,; \qquad w \longrightarrow e^{-2\pi i/3}w \tag{C.2}$$

which preserves $\mathcal{N}=1$ supersymmetry on the D3-brane world-volume. The geometric action is reflected in the field theory as

$$A_1 \longrightarrow e^{\pi i}A_1\,; \qquad B_1 \longrightarrow e^{-\pi i/3}B_1\,; \qquad A_2 \longrightarrow e^{-\pi i/3}A_2\,; \qquad B_2 \longrightarrow e^{-\pi i/3}B_2\,. \tag{C.3}$$

Finally, the action of θ may be embedded on the $\mathrm{U}(N_1)$, $\mathrm{U}(N_2)$ gauge degrees of freedom, through the matrices

$$\gamma^{(1)}_{\theta,3} = \mathrm{diag}\left(I_{n_0}, e^{2\pi i\frac{1}{3}}I_{n_1}, e^{2\pi i\frac{2}{3}}I_{n_2}\right)\,; \qquad \gamma^{(2)}_{\theta,3} = \mathrm{diag}\left(e^{\pi i}I_{n'_0}, e^{\pi i\frac{5}{3}}I_{n'_1}, e^{\pi i\frac{1}{3}}I_{n'_2}\right)\,. \tag{C.4}$$

[18] Alternative approaches include the direct construction of field theories with the right symmetries and the correct moduli space [69, 70], and the use of T-duality [71, 72] to configurations of NS-branes and D4-branes [73, 58].

The field theory on D3-branes at the $\mathbb{Z}_3$ orbifold of the conifold is obtained by projecting the conifold field theory onto states invariant under the combined (geometric plus Chan-Paton) action of θ. The resulting spectrum is

$$\begin{array}{llll} & \mathrm{U}(n_0)\times \mathrm{U}(n_0')\times \mathrm{U}(n_1)\times \mathrm{U}(n_1')\times \mathrm{U}(n_2)\times \mathrm{U}(n_2')\,, & & \\ a_0:(n_0,\overline{n}_0')\,, & b_2:(n_2,\overline{n}_0')\,, & c_0:(\overline{n}_1,n_0')\,, & d_0:(\overline{n}_1,n_0')\,, \\ a_1:(n_1,\overline{n}_1')\,, & b_0:(n_0,\overline{n}_1')\,, & c_1:(\overline{n}_2,n_1')\,, & d_1:(\overline{n}_2,n_1')\,, \\ a_2:(n_1,\overline{n}_1')\,, & b_1:(n_1,\overline{n}_2')\,, & c_2:(\overline{n}_0,n_2')\,, & d_2:(\overline{n}_0,n_2') \end{array} \tag{C.5}$$

with superpotential

$$W=\mathrm{Tr}\left[a_0c_0b_1d_2-a_0d_0b_1c_2+a_1c_1b_2d_0-a_1d_1b_2c_0+a_2c_2b_0d_1-a_2d_2b_0c_1\right]. \tag{C.6}$$

In general the gauge anomalies in this theory are non vanishing, but a suitable set of D7-branes can be introduced to yield the configuration consistent.

Assuming a 37 sector with an arbitrary set of bifundamental representations, it is possible to perform the analysis of U(1) anomalies. In general, mixed anomalies do not vanish, but have a nice factorized structure which suggests they are cancelled through a Green-Schwarz mechanism mediated by closed string twisted modes, generalizing the observation in [24] to these non-orbifold singularities. We also conclude that the only non-anomalous U(1) in this example is the diagonal combination $Q_{\rm diag}=\sum_{i=0}^{2}(Q_{n_i}/n_i+Q_{n_i'}/n_i')$.

The singularity leading to field theories with triplication of at least some chiral multiplet is obtained as a partial blow-up of this $\mathbb{Z}_3$ quotient of the conifold. The partial resolution is manifested in the field theory as an F-flat direction, which is not D-flat by itself, but requires turning on a Fayet-Illiopoulos term, i.e. turning on a vev for blow-up modes. For instance, when $n_0=n_0'$, $n_2=n_2'$ there is one such direction corresponding to diagonal vevs $\langle a_0\rangle=v_0$, $\langle a_2\rangle=v_2$, along which the field theory is reduced to

$$\begin{array}{lll} & \mathrm{U}(n_0)\times \mathrm{U}(n_1)\times \mathrm{U}(n_1')\times \mathrm{U}(n_2)\,, & \\ a_1:(n_1,\overline{n}_1')\,; & b_0:(n_0,\overline{n}_1')\,; & c_0,d_0:(\overline{n}_1,n_0)\,, \\ b_1:(n_1,\overline{n}_2)\,; & c_1,d_1:(\overline{n}_2,n_1')\,; & b_2,c_2,d_2:(n_2,\overline{n}_0) \end{array} \tag{C.7}$$

with superpotential

$$W=v_0\,\mathrm{Tr}(c_0b_1d_2-d_0b_1c_2)+\mathrm{Tr}(a_1c_1b_2d_0-a_1d_1b_2c_0)+v_2\,\mathrm{Tr}(c_2b_0d_1-d_2b_0c_1)\,. \tag{C.8}$$

This corresponds to a partial blow-up to a certain singular variety X, whose form could be determined from the above choice of expectation values. It is possible to follow the above construction in the presence of D7-branes, but since the computation is rather involved, we just quote the result. One may introduce four kinds of D7-branes, containing a total of ten unitary group factors, with ranks denoted u_1, and v_i, w_i, x_i. The 37, 73 sectors are

$$\begin{array}{ll}
(n_1', \overline{u}_1)\,,\ (u_1, \overline{n}_1) \longrightarrow a_1\,; & (n_1', \overline{v}_1)\,,\ (v_1, \overline{n}_0) \longrightarrow b_0\,, \\
(n_2, \overline{v}_2)\,,\ (v_2, \overline{n}_1) \longrightarrow b_1\,; & (n_0, \overline{v}_3)\,,\ (v_3, \overline{n}_2) \longrightarrow b_2\,, \\
(n_1, \overline{w}_1)\,,\ (w_1, \overline{n}_0) \longrightarrow c_0\,; & (n_2, \overline{w}_2)\,,\ (w_2, \overline{n}_1') \longrightarrow c_1\,, \\
(n_0, \overline{w}_3)\,,\ (w_3, \overline{n}_2) \longrightarrow c_2\,; & (n_1, \overline{x}_1)\,,\ (x_1, \overline{n}_0) \longrightarrow d_0\,, \\
(n_2, \overline{x}_2)\,,\ (x_2, \overline{n}_1') \longrightarrow d_1\,; & (n_0, \overline{x}_3)\,,\ (x_3, \overline{n}_2) \longrightarrow d_2
\end{array} \qquad (C.9)$$

we have also indicated by an arrow the 33 field to which the corresponding 37, 73 fields couple in the superpotential. Tadpole conditions cannot be computed directly, but we assume they amount to cancellation of non-abelian anomalies. This should also guarantee that U(1) anomalies cancel by the Green-Scharz mechanism. In this case there are two non-anomalous U(1)'s generated by

$$Q_{\rm diag} = \frac{Q_{n_0}}{n_0} + \frac{Q_{n_1}}{n_1} + \frac{Q_{n_1'}}{n_1'} + \frac{Q_{n2}}{n_2}\,; \qquad Q' = \frac{2}{n_1} Q_{n_1} - \frac{Q_{n_1'}}{n_1'} - \frac{Q_{n2}}{n_2}\,. \qquad (C.10)$$

When $n_1 = n_1'$, a further diagonal vev $\langle a_1 \rangle = v_1$ corresponds to a blow-up of X to the $\mathbb{C}^3/\mathbb{Z}_3$ singularity. Indeed, the resulting field theory agrees with (3.7), after the replacements $b_i, c_i, c_i \to \Phi^r_{i,i+1}$, $v_i, w_i, x_i \to u^r_i$. The 33 piece of the superpotential

$$W = v_0\,{\rm Tr}(b_1 d_2 c_0 - b_1 c_2 d_0) + v_1\,{\rm Tr}(b_2 d_0 c_1 - b_2 c_0 d_1) + v_2\,{\rm Tr}(b_0 d_1 c_2 - b_0 c_1 d_2) \quad (C.11)$$

is more flexible than the 33 piece in (3.8), a fact that may have phenomenological applications.

D. Non-supersymmetric models from antibranes

The computation of the spectrum in a set of D3/$\overline{\rm D3}$-branes at orbifold singularities in the presence of D7/$\overline{\rm D7}$-branes can be extracted from [13] (for a recent review on non-supersymmetric brane configurations with references to previous material see [78]). The only differences with respect to the case without antibranes is the opposite GSO projections in the brane-antibrane sectors. For instance, in $\bar{\mathbf{3}}\bar{\mathbf{3}}$ sectors, GSO projection is as usual and the spectrum is analogous to that in 33 sectors.

We consider a generically non-SUSY $\mathbb{Z}_N$ singularity. Let us introduce D3, $\overline{\rm D3}$ Chan-Paton embeddings $\gamma_{\theta,3}$, $\gamma_{\theta,\bar{3}}$ with n_j, m_j eigenvalues $e^{2\pi i j/N}$. Consider the $\mathbf{3}\bar{\mathbf{3}} + \bar{\mathbf{3}}\mathbf{3}$ sector. In the NS sector the complex tachyon, $\lambda|0\rangle$ survives the GSO, whereas the massless states $\Psi_{-1/2}|0\rangle$ do not. The orbifold projection on the Chan-Paton factors for the tachyons is

$$\lambda = \gamma_{\theta,3} \lambda \gamma_{\theta,\bar{3}}^{-1}\,, \qquad \lambda = \gamma_{\theta,\bar{3}} \lambda \gamma_{\theta,3}^{-1} \qquad (D.1)$$

leading to complex tachyons in the representation $\sum_{i=0}^{N-1}\,[(n_i,\overline{m}_i)+(m_i,\overline{n}_i)]$. In the R sector, we obtain states $\lambda|s_1,s_2,s_3,s_4\rangle$, with $s_i=\pm 1/2$ and $\sum_i s_i=$ even. We get *right*-handed spacetime fermions $s_4=1/2$ with the projection

$$\lambda_{3\bar{3}}=e^{2\pi i a_\alpha/N}\gamma_{\theta,3}\lambda_{3\bar{3}}\gamma_{\theta,\bar{3}}^{-1}\,,\qquad \lambda_{\bar{3}3}=e^{2\pi i a_\alpha/N}\gamma_{\theta,\bar{3}}\lambda_{\bar{3}3}^{(a)}\gamma_{\theta,3}^{-1}\,. \tag{D.2}$$

They transform in the representation $\sum_{i=0}^{N-1}[(n_i,\overline{m}_{i+a_\alpha})+(m_i,\overline{n}_{i+a_\alpha})]$. The $\mathbf{3\bar{7}_r}+\mathbf{\bar{7}_r 3}$ and $\mathbf{\bar{3}7_r}+\mathbf{7_r\bar{3}}$ sectors are analogous to the $\mathbf{37_i}+\mathbf{7_i 3}$ sector, with a few modifications due only to the opposite GSO projections. Consider introducing $\overline{\mathrm{D}7}_3$-branes, with $\gamma_{\theta,\bar{7}_3}$ having v_j eigenvalues $e^{2\pi i j/N}$, and let us center on the case $b_3=$ even. Scalars arise from the NS sector, which has fermion zero modes along the DN directions, Y_1, Y_2. Massless states have the form $\lambda|s_1,s_2\rangle$, with $s_1=-s_2=\pm 1/2$. The orbifold projection for $|1/2,1/2\rangle$ gives

$$\lambda_{3\bar{7}_3}=e^{-2\pi i(b_1-b_2)/2}\gamma_{\theta,3}\lambda_{3\bar{7}_3}\gamma_{\theta,\bar{7}_3}^{-1}\,,\qquad \lambda_{\bar{7}_3 3}=e^{-2\pi i(b_1-b_2)/2}\gamma_{\theta,\bar{7}_3}\lambda_{\bar{7}_3 3}\gamma_{\theta,3}^{-1}\,. \tag{D.3}$$

We obtain complex scalars in the representation $\sum_{i=0}^{N-1}\,[\,(\,n_i,\overline{v}_{i+\frac{1}{2}(b_1-b_2)}\,)\,+\,(\,v_i,\overline{n}_{i+\frac{1}{2}(b_1-b_2)}\,)\,]$.

Spacetime fermions arise from the R sector, which has fermion zero modes along the DD and NN directions, Y_3, Y_4. Massless states are $\lambda|s_3;s_4\rangle$, with $s_3=-s_4=\pm 1/2$. The orbifold projection for $s_4=1/2$ gives

$$\lambda_{3\bar{7}_3}=e^{2\pi i b_3/2}\gamma_{\theta,3}\lambda_{3\bar{7}_3}\gamma_{\theta,\bar{7}_3}^{-1}\,,\qquad \lambda_{\bar{7}_3 3}=e^{2\pi i b_3/2}\gamma_{\theta,\bar{7}_3}\lambda_{\bar{7}_3 3}\gamma_{\theta,3}^{-1}\,. \tag{D.4}$$

We obtain *right*-handed fermions in the representation $\sum_{i=0}^{N-1}[(n_i,\overline{v}_{i-\frac{1}{2}b_3})+(v_i,\overline{n}_{i-\frac{1}{2}b_3})]$. The spectrum in the $\bar{3}7_3$, $7_3\bar{3}$ is computed analogously.

The analysis of gauge anomalies in the presence of antibranes can be performed in analogy with that in sections 2.2 and 2.3 [13, 14]. The non-abelian gauge anomalies are equivalent to twisted RR tadpole cancellation conditions, which have the form (2.18) with the replacement $\mathrm{Tr}\,\gamma_{\theta^k,p}\to\mathrm{Tr}\,\gamma_{\theta^k,p}-\mathrm{Tr}\,\gamma_{\theta^k,\bar{p}}$. This reflects the opposite RR charges of branes and antibranes. Concerning U(1) anomalies, one can show the existence of two non-anomalous U(1) per twist with fixed planes, associated to combinations (2.21) for branes and antibranes, respectively.

E. T-duality of the orientifold models

It is well known that T-duality changes Dirichlet and Neumann boundary conditions, therefore T-duality with respect to the three complex planes should map our models, where the standard model is embedded in D3-branes, to models where the standard model arises from D5, D7 or D9-branes. Since some standard-like models from D7- and D9-branes have recently been build [14, 16], it is worth seeing the explicit correspondence between those models and our present constructions. In particular we will show that, in a suitable T-dual version, the realistic sectors in the models in [14, 16] correspond to local structures of D3- and D7-branes at $\mathbb{C}^3/\mathbb{Z}_3$ singular points, of the type studied in section 3. Hence, our results "explain" the natural appearance of certain features (like hypercharge or $B{-}L$ U(1) gauge factors, or three generations) in the models in [14, 16].

Dual of the Left-Right model of [14]. Let us start with the simplest model in [14, 16], a Left-Right symmetric model constructed from a type-IIB orientifold with D7_3-branes at $Y_3 = 0$, and D3, $\overline{\text{D3}}$-branes at some of the fixed points. Since the D7-branes sit at the orientifold plane O7, the twist matrices $\gamma_{\theta,7}$ and $\gamma_{\theta,3}$ have to satisfy the tadpole condition

$$\mathrm{Tr}\,\gamma_7 + 3\,(\mathrm{Tr}\,\gamma_3 - \mathrm{Tr}\,\gamma_{\bar{3}}) = -4\,. \tag{E.1}$$

Taking $\gamma_7 = (\tilde{\gamma}_7, \tilde{\gamma}_7^*)$ with

$$\tilde{\gamma}_7 = \mathrm{diag}\left(\alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_2, \mathrm{I}_2; \mathrm{I}_2, \alpha \mathrm{I}_7\right) \tag{E.2}$$

and Wilson line in the first complex plane $\tilde{\gamma}_W = \mathrm{diag}(\alpha \mathrm{I}_7, \mathrm{I}_9)$, the gauge group is $\mathrm{U}(3) \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R \times [\mathrm{SO}(4) \times \mathrm{U}(7)]$. To satisfy the tadpole constraint, we just need to add D3-branes at the three points $(-1, m, 0)$ with $\gamma_3 = \mathrm{diag}(\alpha, \alpha^2)$ (since $\mathrm{Tr}\,\gamma_7\gamma_W^2 = -1$). As shown in [14, 16], this is a three-family model. Additional $\overline{\text{D3}}$-branes must be introduced to cancel the untwisted tadpole, but we will not discuss them in detail here.

To find the dual of this model in the approach of the present paper we have to dualize with respect to the first two complex planes so that the D7_3-branes become D3-branes and viceversa. Also under T-duality Wilson lines become displacement of the branes, although the explicit mapping is rather involved. The best strategy in the present case is to construct the model explicitly.

Since the Wilson lines above only affect the 14 entries leading to the LR model, these correspond to the D3-branes that sit outside the origin in the T-dual version. Therefore we consider 18 D3-branes at the origin, with twist matrix $\gamma_{\theta,3} = (\tilde{\gamma}_3, \tilde{\gamma}_3^*)$ and

$$\tilde{\gamma}_3 = \mathrm{diag}\left(\mathrm{I}_2, \alpha \mathrm{I}_7\right). \tag{E.3}$$

This should correspond to the "hidden" part of the original model (last entries in (E.2), and indeed it leads to an $\mathrm{SO}(4) \times \mathrm{U}(7)$ gauge group. Since the origin is an O3-plane, the tadpole condition is (4.14). Since $\mathrm{Tr}\,\gamma_3 = -3$ we can cancel tadpoles introducing six D7-branes at $Y_3 = 0$ with $\gamma_{\theta,7} = \mathrm{diag}(\alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_3)$.

After introducing these D7-branes, we have to worry about tadpole cancellation at the other fixed points in the Y_1, Y_2 planes, given by (4.15). At the points $(\pm 1, 0, 0)$ (mapped into each other under the orientifold action $(-1)^F \Omega R_1 R_2 R_3$) we can put seven D3-branes on each, with twist matrix

$$\gamma_{\theta,3} = \mathrm{diag}\left(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2\right) \tag{E.4}$$

giving rise to a three-generation $\mathrm{SU}(3) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L}$ LR model of the type studied in section 3.4.

Notice that we have introduced 18 D3-branes at the origin and $7 + 7 = 14$ at the $(\pm 1, 0, 0)$ points, already saturating the total number of allowed D3-branes (32). Therefore, twisted tadpoles at the remaining fixed points should cancel without additional D3-branes. This can be achieved by introducing a Wilson line in the Y_2 direction (hence not affecting the points $(m, 0, 0)$) , such that $\mathrm{Tr}\, \gamma_7 \gamma_W = \mathrm{Tr}\, \gamma_7 \gamma_W^2 = 0$. The right choice is

$$\gamma_W = \mathrm{diag}\left(1, \alpha, \alpha^2; 1, \alpha^2, \alpha\right). \tag{E.5}$$

Therefore we do not have to introduce D3-branes at any of the six fixed points $(m, \pm 1, 0)$. Notice also that this Wilson line breaks the D7-brane gauge group U(3) to $\mathrm{U}(1)^3$, as in the original model. The appearance of this Wilson line in the dual model reflects the fact that in the original model the six D3-branes were distributed among three different fixed points.

The last consistency requirement is to have an equal number of D7- and $\overline{\mathrm{D7}}$-branes. To satisfy it, we put three anti-D7-branes with $\gamma_{\bar{7}} = (1, \alpha, \alpha^2)$ at the two mirror locations $Y_3 = \pm 1$. Since this matrix is traceless there is no need to add more D3-branes to cancel tadpoles. These anti-branes can move freely in the Y_3 direction. Moreover, it is possible to turn on continuous Wilson lines along their world-volume, i.e. the directions Y_1, Y_2. This corresponds to possibility of moving the $\overline{\mathrm{D3}}$-branes in the original model in the three complex dimensions of the bulk. They are naturally dual to the anti D3-branes of the original model.

A variant Left-Right orientifold model and its dual. Let us consider a variant Left-Right orientifold model slightly different from the one in section 4. We satisfy the tadpole equations (4.14) at the origin by having 12 D7-branes and 4 D3-branes with:

$$\gamma_7 = \left(\alpha \mathrm{I}_6, \alpha^2 \mathrm{I}_6\right), \qquad \gamma_3 = \left(\alpha, \alpha^2, \alpha, \alpha^2\right). \tag{E.6}$$

In the second plane, we add a Wilson line:

$$\gamma_W = \left(1, \alpha, \alpha^2, \mathrm{I}_3, 1, \alpha, \alpha^2, \mathrm{I}_3\right). \tag{E.7}$$

Since $\mathrm{Tr}\, \gamma_7 \gamma_W = \mathrm{Tr}\, \gamma_7 \gamma_W^2 = -3$ we can put a Left-Right model at each of the points $(0, \pm 1, 0)$ in terms of 7 D3-branes with

$$\gamma_3 = \left(\mathrm{I}_3, \alpha \mathrm{I}_2, \alpha^2 \mathrm{I}_2\right) \tag{E.8}$$

satisfying the tadpole condition (4.15). In order to minimize the number of D3-branes needed to cancel tadpoles (so we do not exceed 32) it is convenient to add a second Wilson line now in the third plane:

$$\gamma'_W = \left(\mathrm{I}_3, 1, \alpha, \alpha^2, \mathrm{I}_3, 1, \alpha, \alpha^2\right). \tag{E.9}$$

Since $\mathrm{Tr}\,\gamma_7\gamma'_W = \mathrm{Tr}\,\gamma_7\gamma'^2_W = -3$ we can cancel tadpoles at the points $(0, 0, \pm 1)$ by putting just one single D3-brane on each with $\gamma_3 = 1$. The advantage of adding the second Wilson line is that at the points $(0, \pm 1, \pm 1)$ there is no need to add any D3-brane because $\mathrm{Tr}\,\gamma_7\gamma^m_W\gamma'^n_W = 0$ for $m, n = 1, 2$. Therefore we have at the plane $Y_1 = 0$ 12 D7-branes and a total of $4 + 7 + 7 + 1 + 1 = 20$ D3-branes.

At each of the planes $Y_1 = \pm 1$ we have to put 6 $\overline{\mathrm{D7}}$-branes and 6 D3-branes distributed on the fixed points $(\pm 1, m, n)$. We achieve tadpole cancellation by having:

$$\gamma_{\overline{7}} = \left(\alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_3\right), \qquad \gamma_W = \left(1, \alpha, \alpha^2, 1, \alpha, \alpha^2\right). \tag{E.10}$$

So we have $\mathrm{Tr}\,\gamma_{\overline{7}} = -3$ and can cancel tadpoles at the points $(\pm 1, 0, m)$ adding 2 D3-branes at each point with $\gamma_3 = (\alpha, \alpha^2)$. This saturates the number of D3-branes $(20 + 2 \times 6 = 32)$ which is OK since at the points $(\pm 1, \pm 1, m)$ there is no need to add D3-branes since $\mathrm{Tr}\,\gamma_{\overline{7}}\gamma_W = \mathrm{Tr}\,\gamma_{\overline{7}}\gamma^2_W = 0$.

We can easily see that the gauge group coming from the D3-branes is $[\mathrm{U}(3) \times \mathrm{U}(2)_L \times \mathrm{U}(2)_R] \times \mathrm{U}(2) \times \mathrm{U}(1)^7$ whereas from the D7-branes is simply $\mathrm{U}(1)^{12}$. The full spectrum may be computed following the same lines as the Left-Right model of section 4.

In order to find the T-dual of this model, in the formalism of [14] we know that at the plane $Y_1 = 0$ we should have 20 D7-branes and 12 D3-branes. To obtain the same gauge group as the model above we choose:

$$\tilde{\gamma}_7 = \left(\alpha \mathrm{I}_3, \alpha^2 \mathrm{I}_2, \mathrm{I}_2, \alpha \mathrm{I}_2, \alpha^2\right), \qquad \gamma_W = \left(\alpha \mathrm{I}_7, \mathrm{I}_2, \alpha\right). \tag{E.11}$$

Since $\mathrm{Tr}\,\gamma_7 = -4$ and $\mathrm{Tr}\,\gamma_7\gamma_W = \mathrm{Tr}\,\gamma_7\gamma^2_W = -1$ we cancel tadpoles by having 2 D3-branes at each of the 6 points $(0, \pm 1, m)$ with $\gamma_3 = (\alpha, \alpha^2)$.

At each of the planes $Y_1 = \pm 1$ we put 6 D7-branes with:

$$\tilde{\gamma}_7 = \alpha \mathrm{I}_3, \qquad \tilde{\gamma}_W = \left(1, \alpha, \alpha^2\right). \tag{E.12}$$

Since $\mathrm{Tr}\,\gamma_7 = -3$ and $\mathrm{Tr}\,\gamma_7\gamma_W = \mathrm{Tr}\,\gamma_7\gamma^2_W = 0$ we cancel tadpoles by adding 2 anti D3-branes at each of the points $(\pm 1, 0, n)$.

We can then see that there is an explicit D3/D7 duality among the two models indicating that they are T-dual of each other. In both cases anti-branes are trapped, as in the orientifold models of section 4.2 providing good examples of gravity mediated supersymmetry breaking. One of the positive points about this Left-Right model is that the extra gauge symmetries are essentially abelian, and most of the U(1) symmetries get broken by the Green-Schwarz mechanism.

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